The Viking Battle - Part 1 2023

Problem 1   Let $k \geq 2$ be an interger. Find the smallest integer $n \geq k + 1$ with the property that there exists a set of n distinct real numbers such that each of its elements can be written as a sum of k other distinct elements of the set.

Problem 2   Let P be the set of all primes. Find all positive integers n such that $n!$ divides

$$\prod_{\substack{p < q \leq n \\ p,q \in P}} (p + q).$$

Problem 3   Let n be a positive integer. We start with n piles of pebbles, each initially containing a single pebble. One can perform moves of the following form: Choose two piles, take an equal number of pebbles from each pile and form a new pile out of these pebbles. For each positive integer n, find the smallest number of non-empty piles that one can obtain by performing a finite sequence of moves of this form.