The Viking Battle - Part 1 2022

Problem 1 Let $ABCD$ be a parallelogram such that $AC = BC$. A point P is chosen on the extension of the segment AB beyond B. The circumcircle of the triangle ACD meets the segment PD again at Q, and the circumcircle of the triangle APQ meets the segment PC again in R. Prove that the lines CD, AQ and BR are concurrent.

Problem 2 Alice is given a rational number $r > 1$ and a line with two points $\mathcal{B} \neq \mathcal{R}$, where point $\mathcal{R}$ contains a red bead and point $\mathcal{B}$ contains a blue bead. Alice plays a solitary game by performing a sequence of moves. In every move, she chooses a (not necessarily) positive integer k, and a bead to move. If that bead is placed at point X, and the other bead is placed at point Y, then Alice moves the chosen bead to point X' with $\overrightarrow{YX'} = r^k\overrightarrow{YX}$.

Alice's goal is to move the red bead to the point $\mathcal{B}$. Find all rational numbers $r > 1$ such that Alice can reach her goal in at most 2021 moves.

Problem 3 A hunter and a rabbit play a game on an infinite grid. First the hunter fixes a colouring of the cells with finitely many colours. The rabbit then secretly chooses a cell to start in. Every minute, the rabbit reports the colour of its current cell to the hunter, and then secretly moves to an adjacent cell that it has not visited before (two cells are adjacent if they share a side). The hunter wins if after some finite time either

- the rabbit cannot move; or
- the hunter can determine the cell in which the rabbit started.

Decide whether there exits a winning strategy for the hunter.