The Viking Battle - Part 1 2019

Problem 1 Let $n \geq 3$ be an integer. Prove that there exists a set S of $2n$ positive integers satisfying the following property: For every $m = 2, 3, \ldots, n$ the set S can be partitioned into two subsets such that one of these subsets has cardinality m and the sums of the elements in each subset are the same.

Problem 2 Let ABC be a triangle with $AB = AC$, and let M be the midpoint of BC. Let P be a point such that $PB < PC$ and PA parallel to BC. Let X and Y be points on the lines PB and PC, respectively, so that B lies on the segment PX, C lies on the segment PY, and $\angle PXM = \angle PYM$. Prove that the quadrilateral $APXY$ is cyclic.

Problem 3 Given any set S of positive integers, show that at least one of the following two assertions holds

1) There exist distinct finite subsets F and G of S such that $\sum_{x \in F} \frac{1}{x} = \sum_{x \in G} \frac{1}{x}$.

2) There exists a positive rational number $r < 1$ such that $\sum_{x \in F} \frac{1}{x} \neq r$ for every finite subset F of S.