Problem 1

$\boxed{7-}$

-1p: does not check/mention that $f \equiv \frac{1}{2}$ is indeed a solution

$\boxed{0+}$

(not additive)

+1p: checks/mentions that $f \equiv \frac{1}{2}$ is a solution

+1p: for $f\left(f\left(f(x)\right)\right) = f(x)$

+2p: shows that $f(1-x) = f(x)$

+3p: for $f(1-x) = f(x)$ and checks that $f \equiv \frac{1}{2}$ is a solution

+3p: for showing that $f \equiv \frac{1}{2}$ on V_f

Problem 2

7-

-0p: missing the empty subsystem; this changes the answer, but makes the problem more difficult

-1p/case: missing some case (e.g. the case when exactly one edge of the cycle connecting to *v* is in *H*)

-2p: correct bijection, but missing proof that H' (or some similar construction) is an even graph; for full mark it suffices to say that the difference for each edge is an even number

0+

(not additive)

+0p: special case(s); but cycle of arbitrary length *n* can give 1 p, if it is translated and solved correctly (see below)

+1p: correct answer only, without any substantial argument

+1p: correct translation into graph theory; e.g. claim that even system is disjoint union of cycles; only picture is not sufficient without naming cycles and/or even degrees

+2p: claim that each town/edge belongs to exactly half the subsystems, equivalently correct size of *S(e)*

+3p: wrong proof that each town/edge is in exactly half the subsystems (e.g. eliminating cycle containing the edge or other wrong constructions of a bijective map, or statement that each cycle belongs to exactly half the subsystems)

Problem 3

Correct answer (only) gives 1p.

The remaining 6 points: 4 points for prime n and 2 points for non-prime n.

In each case half the points are given for correct strategy and the second half for a correct proof that the strategy works. Example of wrong strategy: choose $2x$ if Anton have chosen x. This gives zero points out of 4, because the strategy is wrong (Anton can choose 2 at the first step and 1 at the second).

For prime n a strategy consists of two choices: at each step (one point) and final (second point). Similarly for the proof.

For example, the proof that for non-prime n we can get zero should use that the number n is odd. Simply saying that one chooses n/d or $n-n/d$ gives only one point out of 2, because it should be shown that these numbers are different. Choosing only one divisor d and claiming that the result will be zero mod n gives 1 point (correct strategy, wrong proof). Claiming that the result is not 1 without mention of divisibility of both numbers (n and the power) also gives only one point.

Similarly for the prime case only choosing $n - a$ each time Anton chooses a without any attempt to say why this is possible (n is odd and $n - a$ cannot be chosen in the previous step) gives 1 out of 2 points for the strategy. But if something like pairing is mentioned it gives 2 points.

Citing Fermat/Euler theorem gives no points, the same for the remark that the power is either zero or ± 1 modulo prime. The statement that exactly half gives $+1$ (for prime) still gives zero points if it is not connected with some reasonable strategy.

Finitely many special cases give 0 points.

Infinitely many special cases as p^2, pq can give one point (for the non-prime part).

Problem 4

7-

-1p: for small flaws, e.g. in showing similarity

0+

+2p: for guessing that the point of intersection is the Miquel point of *BCED*

+2p: for guessing that the point of intersection lies on the circumscribed circle of triangle *ADE*