THE GEORG MOHR CONTEST 2022

First round

Tuesday, November 23 2021

Duration: 90 minutes
Aids allowed: none
Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options, A, B, C, D and E. One of these options is the correct answer.

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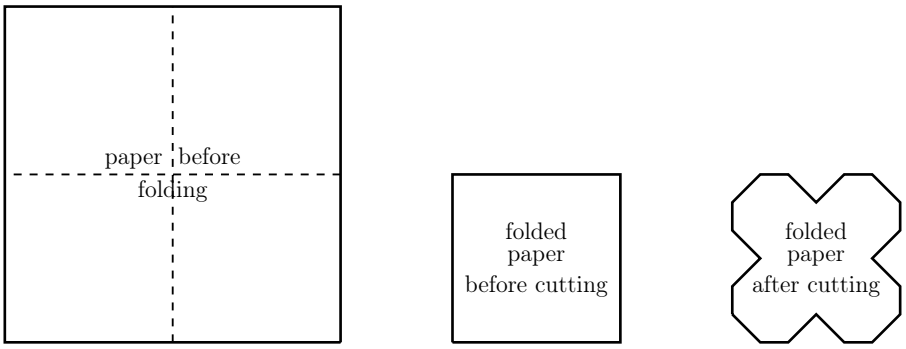
1. Using the three digits A, B and C, three 3-digit numbers ABC, BCA og CAB are formed whose sum is 2331.

   What is $A + B + C$?

$$\begin{array}{r} A\ B\ C \\ +\ B\ C\ A \\ +\ C\ A\ B \\ \hline 2\ 3\ 3\ 1 \end{array}$$

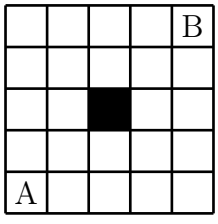
A) 11 B) 13 C) 21 D) 22 E) 31

2. A square piece of paper is folded in the middle twice. In the folded paper eight cuts are made along the edge as shown in the figure. How many holes are there in the paper when it is unfolded?



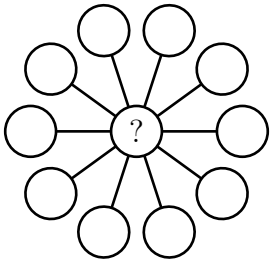
A) 2 B) 4 C) 5 D) 6 E) 8

3. Emma must walk from A to B by moving from tile to tile. She must move alternately horizontally and vertically, she cannot use the same time multiple times, and the black tile must not be used at all. How many possible routes are there from A to B?



A) 4 B) 5 C) 6 D) 8 E) 10

4. Selma wants to write a number in each of the 11 circles such that the sum of the numbers in the 10 outer circles is 1000, and such that the sum of the three numbers in each of the five diagonals is also 1000. Which number must she write in the middle circle?



A) 100 B) 300 C) 500 D) 800 E) there are several choices

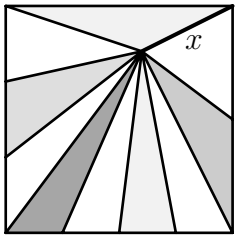
5. Karen has chosen two positive numbers a and b. The number a is less than 1, and the number b is greater than 1. Which of the following numbers is the greatest?

A) $a + b$ B) $a \cdot b$ C) $\frac{a}{b}$ D) a^{2022} E) b

6. Henrik has both small and large stuffed animals, but only two kinds: bears and cats. Each stuffed animal is either white or brown. All the small stuffed animals are cats, and all the white stuffed animals are bears. Which of the following statements is definitely true?

A) all the brown animals are cats B) all the brown animals are bears
C) all the cats are small D) none of the brown animals are small
E) all the small animals are brown

7. A beautiful square mosaic window consists of 10 triangles having the the same area. The side length of the window is 5. What is the length of the segment marked with x on the figure?

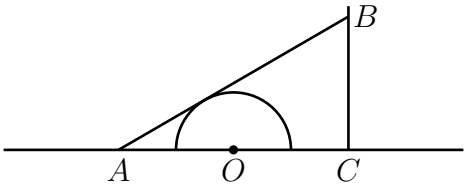


A) $\frac{3\sqrt{2}}{2}$ B) 2 C) $\frac{\pi}{2}$ D) $\frac{21}{10}$ E) $\sqrt{5}$

8. At a common lesson, all 150 1st grade students and 100 2nd grade students at the school handed in a ballot with a yes or no to the question "Are you satisfied with the food in the canteen?" 80 % of the 1st grade students answered yes, while only 40 % of the 2nd grade students answered yes. After the lesson a random ballot was drawn, and it turned out to be a yes. What is the probability it came from a 1st grade student?

A) 60 % B) 88 % C) $\frac{9}{10}$ D) 75 % E) $\frac{2}{3}$

9. The segments AC and BC are orthogonal. The semicircle has radius 1, and its center O lies midway between A and C. The segment AB is tangent to the semicircle. The length of AC is 4. What is the length of BC?



A) 2,5 B) $\frac{\sqrt{5}}{2}$ C) $\frac{4}{\sqrt{3}}$ D) $1+\frac{\sqrt{3}}{2}$ E) $1+\frac{\pi}{2}$

10. When purchasing from Planer & Saw one is given a control number formed in the following way: Starting from an integer between 1 and 100, multiply by 17, add 341, multiply by 4, add 814, and finally multiply by 5. Which of the following numbers is a possible control number?

A) 37410 B) 26705 C) 58900 D) 37786 E) 17825

ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

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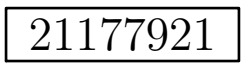
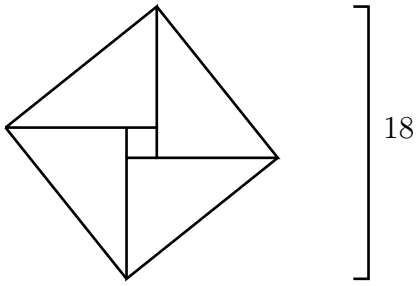
11. The number 21177921 is written on a strip of paper.



Georg cuts the strip into three pieces so that he gets three numbers. Then he adds the three numbers. What is the smallest possible result he can obtain?

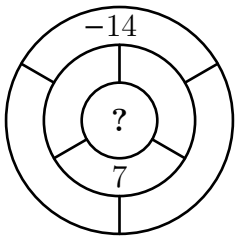
12. The large square is divided into four identical right-angled triangles and a small square. The area of the small square is 4, and the height of the figure is 18. What is the area of the large square?



13. A triangle has three distinct angles, all integers when expressed in degrees. What is the smallest number one can get when adding the largest and smallest angles, expressed in degrees?

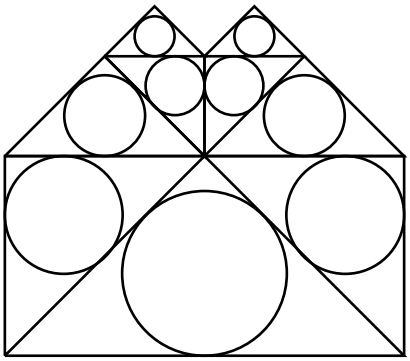
14. The numbers from 1 to 24 are multipled, and the result is called T. What is the smallest positive integer which does not divide T?

15. Each of the seven regions have contained an integer. The number in each region was the sum of the numbers in all the adjacent regions. Most of the numbers have now been erased. Which number was written in the middle region?



16. Alma and Bertha have a piece of string. Alma wants to cut the string into 12 equally long pieces, and she sets marks on the string indicating where to cut to achieve that. Bertha wants to cut the string into 15 equally long pieces, and she also sets marks on the string indicating where to cut to achieve that. After that, they cut the string at all the marks. How many pieces of string do they end up with?

17. The figure shows a decoration on a facade. The decoration consists of right-angled, isosceles triangles, each with an inscribed circle. The largest circle has an area of 8 m^2. What is the total area (in m^2) of all the circles?



18. Both the rows and the columns on a 10×10 board are numbered from 1 to 10. In each square is written the sum of the square's row number and column number. Finally all the numbers in all the squares are added. What is the result?

19. There are two idential hand gel containers at the entrance to CenterKøbmanden. William has noticed that the customers apparently prefer the one to the right, because that one gets emptied in 90 minutes, while the left one lasts for 210 minutes. Instead of just refilling one and then the other when it becomes necessary, he comes up with the idea of interchanging the two containers at an appropriate time. How many minutes after a refill of both containers must he switch them around so that they become empty at the same time?

20. Georg must distribute 100 black marbles and 100 white marbles in 10 urns so that there is at least one marble in each urn. Georg's mother then chooses one of the 10 urns at random and then draws one marble from that urn at random. Georg distributes the marbles so that the probability that his mother draws a white marble is as small as possible. How many marbles does he put in the urn containing most marbles?

Sponsors: Undervisningsministeriet, Novo Nordisk Fonden, Jobindex, Lundbeckfonden, Georg Mohr Fonden og Matematiklærerforeningen.