

Language: English

July 26, 2007

**Problem 4.** In triangle  $ABC$  the bisector of angle  $BCA$  intersects the circumcircle again at  $R$ , the perpendicular bisector of  $BC$  at  $P$ , and the perpendicular bisector of  $AC$  at  $Q$ . The midpoint of  $BC$  is  $K$  and the midpoint of  $AC$  is  $L$ . Prove that the triangles  $RPK$  and  $RQL$  have the same area.

**Problem 5.** Let  $a$  and  $b$  be positive integers. Show that if  $4ab - 1$  divides  $(4a^2 - 1)^2$ , then  $a = b$ .

**Problem 6.** Let  $n$  be a positive integer. Consider

$$S = \{(x, y, z) : x, y, z \in \{0, 1, \dots, n\}, x + y + z > 0\}$$

as a set of  $(n+1)^3 - 1$  points in three-dimensional space. Determine the smallest possible number of planes, the union of which contains  $S$  but does not include  $(0, 0, 0)$ .

*Time allowed: 4 hours 30 minutes  
Each problem is worth 7 points*