The Georg Mohr Contest 2019
Second Round

Tuesday, January 8th, 2019 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. Which positive integers satisfy that the sum of the number's last three digits added to the number itself yields 2029?

Problem 2.

Two distinct numbers a and b satisfy that the two equations

$$x^{2019} + ax + 2b = 0 \quad \text{and} \quad x^{2019} + bx + 2a = 0$$

have a common solution.

Determine all possible values of $a + b$.

Problem 3.

Seven positive integers are written on a piece of paper. No matter which five numbers one chooses, each of the remaining two numbers divides the sum of the five chosen numbers.

How many distinct numbers can there be among the seven?

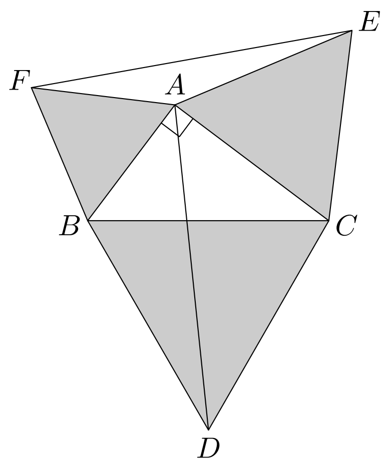
Problem 4.

Georg writes a positive integer a on a blackboard. As long as there is a number on the blackboard, he does the following each day:

- If the last digit in the number on the blackboard is less than or equal to 5, he erases that last digit. (If there is only this digit, the blackboard thus becomes empty.)
- Otherwise he erases the entire number and writes 9 times the number.

Can Georg choose a in such a way that the blackboard never becomes empty?

Problem 5. In the figure below the triangles BCD, CAE and ABF are equilateral, and the triangle ABC is right-angled with $\angle A = 90°$.



Prove that $|AD| = |EF|$.

Sponsors: Undervisningsministeriet, Jobindex, VILLUM FONDEN, Georg Mohr Fonden and Matematiklærerforeningen.