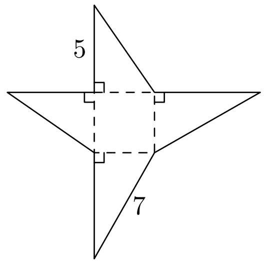
The Georg Mohr Contest 2005

Tuesday 11 January 2005 at 9-13 hours

Tools for writing and drawing are the only ones allowed

Problem 1. This figure is cut out from a sheet of paper. Folding the sides upwards along the dashed lines, one gets a (non-equilateral) pyramid with a quadratic base.



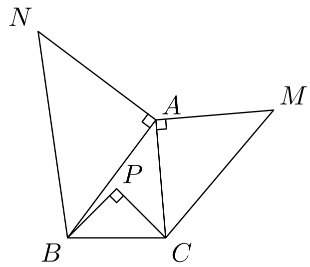
Calculate the area of the base.

Problem 2. For any positive real number a determine the number of solutions (x, y) of the system of equations

$$|x| + |y| = 1\,,$$
$$x^2 + y^2 = a\,,$$

where x and y are real numbers.

Problem 3. The point P lies inside $\triangle ABC$ so that $\triangle BPC$ is isosceles, and angle P is a right angle. Furthermore both $\triangle BAN$ and $\triangle CAM$ are isosceles with a right angle at A, and both are outside $\triangle ABC$.



Show that $\triangle MNP$ is isosceles and right-angled.

Problem 4. Each of fourteen pupils writes an integer number on the blackboard. When meeting later on their mathematics teacher Homer Grog, they tell him that whichever of the numbers on the blackboard they erased, the rest of them could be divided into three goups with equal sums. They tell him, as well, that the numbers on the blackboard were integer numbers.

Is it now possible for Homer Grog to determine which numbers the pupils wrote on the blackboard?

Problem 5. For which real numbers p does the system of equations

$$x_1^4 + \frac{1}{x_1^2} = px_2\,,$$
$$x_2^4 + \frac{1}{x_2^2} = px_3\,,$$
$$\vdots$$
$$x_{2004}^4 + \frac{1}{x_{2004}^2} = px_{2005}\,,$$
$$x_{2005}^4 + \frac{1}{x_{2005}^2} = px_1$$

have just one solution $(x_1, x_2, \ldots, x_{2005})$, where $x_1, x_2, \ldots, x_{2005}$ are real numbers?

Sponsors: Georg Mohr Fonden, Dansk Matematisk Forening, Matematiklærerforeningen, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, UNI-C and Gyldendal.