Baltic Way

14 November 2026, Vilnius, Lithuania

Version: English

Time allowed: 4 h 30 min
Time for questions: 30 min
Tools for writing and drawing are the only ones allowed.

Problem 1 Turbo is jumping on the positive integers in such a way that at time $t \in \mathbb{Z}_{>0}$ Turbo is at the integer a_t. Turbo begins at a_1 and chooses the numbers they visit in such a way that

$$\begin{aligned} a_t \cdot a_{t+1} &\geqslant t + 401 \qquad \text{if } t \text{ is odd,} \\ a_t \cdot a_{t+1} &\leqslant t + 401 \qquad \text{if } t \text{ is even} \end{aligned}$$

for all positive integers t.

Given that no 410 consecutive integers are all visited by Turbo, prove that they visit at least one of the numbers 2026 and 2027.

Problem 2 Find all functions $f : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ such that

$$f(x + 2y + f(x)) = y + f(y + 2x)$$

holds for all $x, y > 0$.

Problem 3 Let $n > 1$ and let $x_1, x_2, \ldots, x_{n+1}$ be positive real numbers with $x_{n+1} = x_1$ satisfying

$$\sum_{i=1}^{n} \frac{x_{i+1}}{x_i} > 2n.$$

Show that

$$\sum_{i=1}^{n} \left(x_i^3 + \frac{1}{x_i^3} \right) > 5n.$$

Problem 4 Find all functions $f : \mathbb{Q} \to \mathbb{R} \setminus \{0\}$ such that

$$(f(x) + f(y) - 1)\, f(x + y) = f(x) f(y)\, (2f(x + y) - 1)$$

holds for all rational numbers x, y.

Problem 5 Let $n \geqslant 3$ be an integer. Determine the smallest real number M such that

$$\frac{x_1 - x_2}{x_1 + x_2} + \frac{x_2 - x_3}{x_2 + x_3} + \cdots + \frac{x_n - x_1}{x_n + x_1} < M$$

holds for all positive real numbers $x_1, x_2, \ldots, x_n$.

Problem 6 A 2027×2027 grid consists of unit squares. Some diagonals of the unit squares are drawn, we call them *walls*. Each unit square contains at most one wall and the top left square contains no walls.

A robot starts from the top left corner of the grid. It starts moving towards the bottom right corner, always moving in straight lines in the direction it is facing. If the robot hits the centre of a unit square containing a wall, it does the following:

- If the wall is perpendicular to its movement direction, it chooses to turn 90 degrees left or right.
- If the wall is parallel to its direction, it keeps moving in the same direction.

The robot stops moving if it meets the edge of the grid.

A unit square is said to be *reachable* if, for some sequence of choices, the robot visits the interior of that square. Find the least number of walls so that at least half of the squares are reachable or prove that no such configuration of walls exists.

Problem 7 Let $n > 1$ be an integer. There are n cards, each of which has a distinct prime written on it. Ann does not know which primes are written on the cards. The cards are placed face down in a row, such that the numbers can not be seen. In a move, Ann picks a set of one or more consecutive cards and is then told the product of the primes on those cards. Determine the least number of moves she must make to determine what prime is written on each card.

Problem 8 A ring of $n > 1$ dancers are dancing to an endless song. Every beat of the song, each dancer swings either their left or their right leg. Each beat, except the first one, they decide which leg to swing according to the following rule: they swing their right leg if their right neighbour swung the same leg as them last beat, and otherwise they swing their left leg. Determine the number of ways the dancers could have swung their legs the first beat if there exists a future beat when everyone swings the same leg as each other.

Note: Two configurations are considered different if there exists a dancer making a different swing in those two configurations.

Problem 9 In the Baltic Garden, every Beetle has a unique string of three digits written on its back. Two Beetles are enemies if the string formed by the first two digits on one is the same as the string formed by the last two digits on the other. A Beetle may be its own enemy.

What is the largest number of Beetles that can live in the garden if no Beetle has an enemy?

Problem 10 Let n be a positive integer. The top right square of a $2n \times 2n$ grid of unit squares is white and all other squares are red. The Wolf performs the following operation some number of times:

1. The Wolf chooses a row or column with an odd number of red squares.
2. The colours in the chosen row or column are changed so that all red squares become white and all white squares become red.

In terms of n, determine the maximum possible number of white squares at the end of the process.

Problem 11 In an acute triangle ABC with $AB < AC$, point D is chosen on side BC. The bisectors of the angles BAD and CAD intersect the circumcircles of triangles ABD and ACD again at points X and Y, respectively. Let N be the midpoint of the arc BC containing A on the circumcircle of triangle ABC. Prove that the points X, A, N, and Y are concyclic.

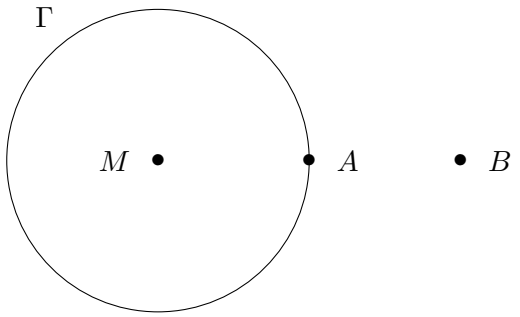
Problem 12 Let ABC be an acute triangle with $AB < AC$. Let O be the circumcentre and Γ be the circumcircle of the triangle ABC. Line AO intersects Γ again at X. Let D be the foot of the altitude from A to BC. Points E and F lie on the segments AC and AB respectively such that $AC \perp DE$ and $AB \perp DF$. Lines DX and EF intersect at L. Line AL intersects Γ again at K. Show that E, F, X, and K are concyclic.

Problem 13 Let the circumcircle of the triangle ABC be Γ and let X be a point on Γ. The points D, E, F are fixed and belong to segments BC, CA, AB, respectively. The midpoints of segments XD, XE and XF are D', E' and F', respectively. Prove that as X varies along Γ, the circumcentre of the triangle $D'E'F'$ moves on a fixed circle.

Problem 14 In a geometry app, a circle Γ centered at M passing through A is displayed. Moreover, a point B is the reflection of M over A. Call points A, B, and M *constructed*. In a move, the user selects two different constructed points P and Q and presses one of two buttons.

- Pressing button 1 displays the line through P and Q.
- Pressing button 2 displays the circle with centre P through Q.

After a button is pressed, all occurring intersection points between all displayed objects are constructed.



Find the smallest number of moves necessary to construct two points $X, Y \in \Gamma$ such that B is the reflection of X over Y.

Problem 15 Let ABC be an acute triangle with circumcentre O. The bisector of angle BAC intersects segment BC at D and the circumcircle of ABC again at M. A circle with centre D and radius DB intersects the line BM again at N and the segment AC at distinct points K and L, where L is between A and K. Lines BL and NK intersect at E, while lines BM and AC intersect at F. Prove that ED and OF are perpendicular.

Problem 16 Let $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ be a multiplicative function such that $f(n) \leqslant n$ for all $n \in \mathbb{Z}_{>0}$. A pair of positive integers (a_1, d) is called *Lithuanian* if the sequence given by $a_{n+1} = f(a_n + d)$ is bounded. Prove that every pair of positive integers is Lithuanian if and only if there are infinitely many prime numbers p for which $f(p^{k_p}) < p^{k_p}$ for some positive integer k_p.

Note: A function $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ is *multiplicative* if $f(mn) = f(m)f(n)$ for all coprime $m, n \in \mathbb{Z}_{>0}$. A sequence $a_1, a_2, \ldots$ is *bounded* if there exists an M such that $|a_n| < M$ for all n.

Problem 17 A positive integer n is *dangerous* if it can be expressed as $n = a \cdot b$, where a, b are positive integers such that $|a - b| < 6$. Does there exist an infinite strictly increasing arithmetic progression of positive integers that contains no dangerous numbers?

Problem 18 Determine all positive integers k for which there exists a positive integer n such that

$$n + k \mid n! + 1.$$

Problem 19 The numbers $1, 2, \ldots, 2026$ are written on a blackboard. A *bad pair* is a pair of integers (a, b) such that $a < b$ and a does not divide b. In a move, Bob chooses a bad pair (a, b) from the blackboard, erases a and b, and writes the remainder of b when divided by a on the blackboard.

Find the minimum number of moves that Bob must make to ensure there are no bad pairs on the blackboard.

Problem 20 Let $n \geqslant 3$. The positive integers $a_1, a_2, \ldots, a_n$ are arranged in a circle so that each integer divides the sum of its two neighbours. Let m be the smallest among $a_1, a_2, \ldots, a_n$. Determine all possible values of

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_n a_1}$$

in terms of n and m.