The Viking Battle - Part 1 2017
Version: English

Problem 1 Let n and k be positive integers where $k < n$. Peter and John play a game where they both know the rules. Peter chooses a secret n-digit binary string. Then he gives John a list of all n-digit binary strings that differ from his secret string in exactly k positions. (For example if $n = 3$, $k = 1$, and the secret string is 101, the list is 001, 111, 100). Now John has to guess the secret string.

What is the minimum number of guesses (in terms of n and k) needed to guarantee the correct answer?

Problem 2 For any positive integer k denote the sum of digits of k in its decimal representation by $S(k)$.

Find all polynomials $P(x)$ with integer coefficients such that for any positive integer $n \geq 2017$ the integer $P(n)$ is positive and

$$S(P(n)) = P(S(n)).$$

Problem 3 Let $B = (-1, 0)$ and $C(1, 0)$ be fixed points in the coordinate plane. A nonempty, bounded subset S of the plan is said to be a *viking* set if

(i) for any triangle $P_1P_2P_3$ there exists a unique point A in S and a permutation σ of the indices $\{1, 2, 3\}$ for which the triangles ABC and $P_{\sigma(1)}P_{\sigma(2)}P_{\sigma(3)}$ are similar; and

(ii) there is a point T in S such that for every Q in S the segment TQ lies entirely in S.

Prove that there exist two distinct viking subsets S and S' of the set

$$\{(x, y) \mid x \geq 0, y \geq 0\}$$

such that if $A \in S$ and $A' \in S'$ are the unique choices of points in (i) then the product $BA \cdot BA'$ is a constant independent of the triangle $P_1P_2P_3$.