The Viking Battle - Part 1 2016
Version: English

Problem 1 Let ABC be an acute triangle with orthocentre H. Let D be the point such that the quadrilateral $HABD$ is a parallelogram (with $AB \parallel HD$ and $AH \parallel BD$). Let E be the point on the line DH such that the line AC passes through the midpoint of the segment HE. Let F be the second point of intersection of the line AC and the circumcircle of triangle DCE.

Prove that $EF = AH$.

Problem 2 The sequence $a_1, a_2, \ldots$ of positive real numbers satisfies

$$a_{k+1} \geqslant \frac{ka_k}{a_k^2 + k - 1}$$

for every positive integer k. Prove that

$$a_1 + a_2 + \cdots + a_n \geqslant n$$

for every $n \geqslant 2$.

Problem 3 Let n be a positive integer. Two players A and B play a game in which they take turns choosing positive integers $k \leq n$. The rules of the game are:

(i) A player cannot choose a number that has been chosen by either player on any previous turn.

(ii) A player cannot choose a number adjacent to any of those the player has already chosen on any previous turn.

(iii) The game is a draw if all numbers have been chosen; otherwise the player who cannot choose a number anymore loses the game.

The player A takes the first turn. Determine the outcome of the game, assuming that both players use optimal strategies.