The 22nd Nordic Mathematical Contest

31 March 2008
English version

Time allowed is 4 hours. Each problem is worth 5 points. The only permitted aids are writing and drawing materials.

Problem 1

Determine all real numbers A, B and C such that there exists a real function f that satisfies

$$f(x + f(y)) = Ax + By + C$$

for all real x and y.

Problem 2

Assume that $n \geq 3$ people with different names sit around a round table. We call any unordered pair of them, say M and N, *dominating*, if

(i) M and N do not sit on adjacent seats, and

(ii) on one (or both) of the arcs connecting M and N along the table edge, all people have names that come alphabetically after the names of M and N.

Determine the minimal number of dominating pairs.

Problem 3

Let ABC be a triangle and let D and E be points on BC and CA, respectively, such that AD and BE are angle bisectors of ABC. Let F and G be points on the circumcircle of ABC such that AF and DE are parallel and FG and BC are parallel. Show that

$$\frac{AG}{BG} = \frac{AB + AC}{AB + BC}.$$

Problem 4

The difference between the cubes of two consecutive positive integers is a square n^2, where n is a positive integer. Show that n is the sum of two squares.