## NMC 2002
### Second draft

**1** A trapezium $ABCD$ with $AB$ and $CD$ parallel and with $AD < CD$ is inscribed in a circle $c$. Furthermore, let $DP$ be a chord parallel to $AC$. Let the tangent to $c$ at $D$ intersect the line $AB$ at $E$ and let $PB$ and $DC$ intersect at $Q$. Show that $EQ = AC$.

2. Let $N$ balls, numbered 1 to $N$, be distributed over two urns. A ball is moved from one urn to the other. We then find that in each urn the mean of the numbers of the balls is increased by the same value, $x$. What is the greatest possible value of $x$?

3. Let $a_1, a_2, \ldots, a_n$, $b_1, b_2, \ldots, b_n$ be real numbers, such that $a_1, a_2, \ldots, a_n$ are all different. If the product

$$(a_i + b_1)(a_i + b_2) \cdot \ldots \cdot (a_i + b_n)$$

takes one and the same value for every $i$, $i = 1, 2, \ldots, n$, show that the product

$$(a_1 + b_j)(a_2 + b_j) \cdot \ldots \cdot (a_n + b_j)$$

also takes one and the same value for every $j$, $j = 1, 2, \ldots, n$.

4. Eva, Per och Anna are playing with their pocket calculators. They select different integers and check if they are divisible by 11 or not. Only 9-digit numbers with all the digits represented are studied. Anna claims that, if such a number is chosen randomly, the probability that the number is a multiple of 11 is exactly $1/11$. Eva does not agree; she believes that the probability is less than $1/11$. Per, on the other side, believes that the probability is greater than $1/11$. Which one is correct?