The 25th Nordic Mathematical Contest

Monday 4 April 2011

English version

The time allowed is 4 hours. Each problem is worth 5 points. The only aids permitted are writing and drawing tools.

Problem 1

When $a_0, a_1, \ldots, a_{1000}$ denote digits, can the sum of the 1001-digit numbers $a_0a_1 \ldots a_{1000}$ and $a_{1000}a_{999} \ldots a_0$ have odd digits only?

Problem 2

In a triangle ABC assume $AB = AC$, and let D and E be points on the extension of segment BA beyond A and on the segment BC, respectively, such that the lines CD and AE are parallel. Prove $CD \geq \frac{4h}{BC} CE$, where h is the height from A in triangle ABC. When does equality hold?

Problem 3

Find all functions f such that

$$f(f(x) + y) = f(x^2 - y) + 4yf(x)$$

for all real numbers x and y.

Problem 4

Show that for any integer $n \geq 2$ the sum of the fractions $\frac{1}{ab}$, where a and b are relatively prime positive integers such that $a < b \leq n$ and $a + b > n$, equals $\frac{1}{2}$.
(Integers a and b are called *relatively prime* if the greatest common divisor of a and b is 1.)