20^{th} Nordic Mathematical Contest

Thursday March 30, 2006

English version

Time allowed: 4 hours. Each problem is worth 5 points.

Problem 1. Let B and C be points on two fixed rays emanating from a point A such that $AB + AC$ is constant.
Prove that there exists a point $D \neq A$ such that the circumcircles of the triangels ABC pass through D for every choice of B and C.

Problem 2. The real numbers x, y and z are not all equal and fulfill

$$x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x} = k$$

Determine all possible values of k.

Problem 3. A sequence of positive integers $\{a_n\}$ is given by

$$a_0 = m \qquad \text{and} \qquad a_{n+1} = a_n^5 + 487 \quad \text{for all} \quad n \geq 0$$

Determine all values of m for which the sequence contains as many square numbers as possible.

Problem 4. The squares of a 100×100 chessboard are painted with 100 different colours. Each square has only one colour and every colour is used exactly 100 times.
Show that there exists a row or a column on the chessboard in which at least 10 colours are used.

Only writing and drawing sets are allowed