

THE GEORG MOHR CONTEST 2021

First round

Tuesday, November 17 2020

Duration: 90 minutes

Aids allowed: none

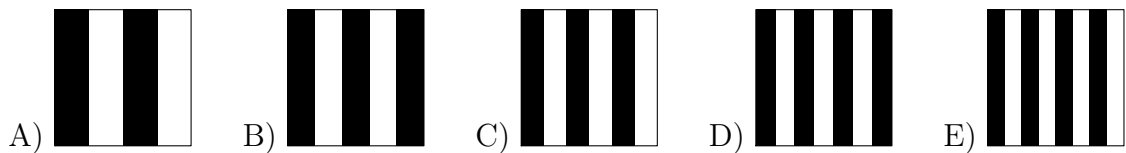
Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options, A, B, C, D and E.
One of these options is the correct answer.

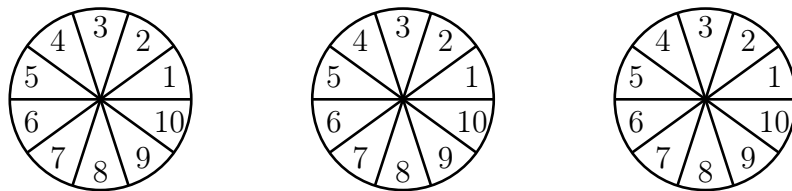
1. The five squares have the same area, and in each square the stripes are equally wide. In which square is the area of the black stripes the largest?



2. Which of the following numbers does 5 *not* divide for any integer n ?

A) $n^2 - 1$ B) $2n + 1$ C) $n \cdot (n + 1)$ D) $10n + 1$ E) $n^3 - 1$

3. Three wheels of fortune with the numbers from 1 to 10 are set in motion. What is the probability that they stop at the same number?



A) $\frac{3}{10}$ B) $(\frac{1}{10})^3$ C) $\frac{10 \cdot 9 \cdot 8}{1000}$ D) $\frac{1}{100}$ E) $\frac{3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8}$

4. Rebecca has drawn a straight line through the point $(4, 10)$ in a coordinate system. The line is given by the equation $y = a \cdot x + b$. Now she wishes to change the line. She adds 3 to a . What does she need to do to b if she wants the changed line to still pass through the point $(4, 10)$?

A) subtract 3 from b B) add 10 to b C) subtract 12 from b
D) subtract 4 from b E) add 3 to b

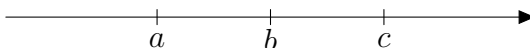
5. Oscar has six pretty toy coins: a 1-krone, a 2-krone, a 4-krone, a 8-krone, a 16-krone and a 32-krone. With those he can pay any amount up to 63 kroner exactly. One day, one of the coins has been lost. This means that he can no longer pay neither 22 kroner nor 45 kroner exactly. Which of the coins has been lost?

- A) the 2-krone B) the 4-krone C) the 8-krone
D) the 16-krone E) it cannot be determined

6. What is the last three digits in the number $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \dots \cdot 2021}{2^{1010}}$?

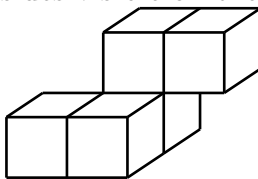
- A) 010 B) 256 C) 125 D) 021 E) 000

7. The three numbers a , b and c lie on the number line as shown with b precisely half-way between a and c :



What can one know with certainty?

- A) $a + b > c$ B) $a + b < c$ C) $a > b \cdot c$ D) $a < b \cdot c$ E) none of the preceding
8. Five completely identical cubes each have two white sides opposite each other and four black sides. The five cubes must be glued together as shown. The gluing must be done in such a way that sides of the same color are glued together. What is the largest possible number of black sides visible on the finished figure?

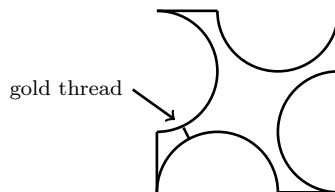


- A) 14 B) 16 C) 18 D) 20 E) 22

9. For every integer n we put $T(n) = 2n - 1$ and $S(n) = n + 4$. For a certain integer n , $S(T(S(T(S(T(n))))))$ is one of the numbers below. Which one?

- A) 518 B) 624 C) 641 D) 821 E) 906

10. The figure, consisting of line segments of length 1 cm and halfcircles with radius 1cm, shows a small piece of silver jewelry with four “arms”. On top of one of the arms there must be placed a piece of gold thread across the arm at the most narrow place. What is the length of this thread?

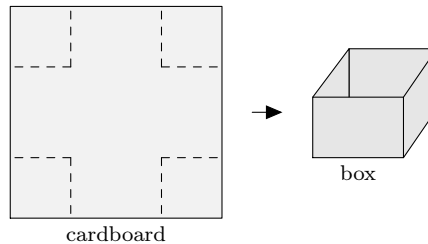


- A) $\sqrt{5} - 2$ cm B) $\frac{\pi}{8}$ cm C) $\frac{1}{4}$ cm D) $2 - \sqrt{3}$ cm E) $\frac{\sqrt{2}}{3}$ cm

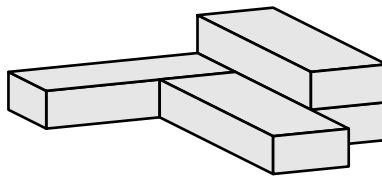
ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

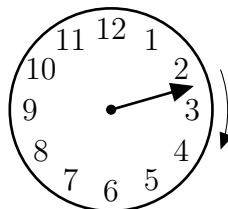
11. Omar has a square piece of cardboard. When he cuts a square with side length 2 cm off each corner and folds the cardboard to a box, the volume of the box is 98 cm^3 . What will the volume measured in cm^3 of the box be if he instead cuts a square with side length 3 cm off each corner?



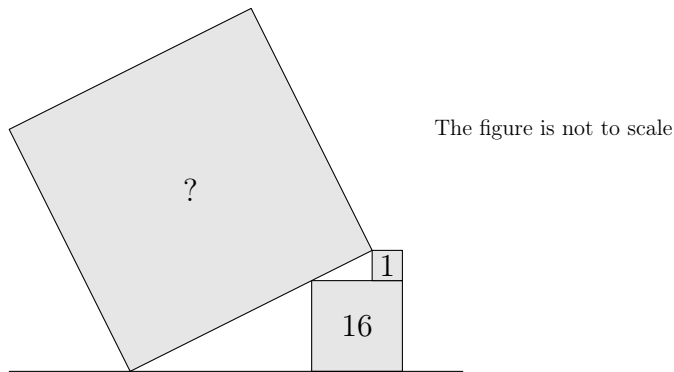
12. Four identical bricks are glued together as shown. One needs 11 grams of paint to paint one of the bricks shown on all sides. How many grams of paint are needed to paint all surfaces on the glued construction?



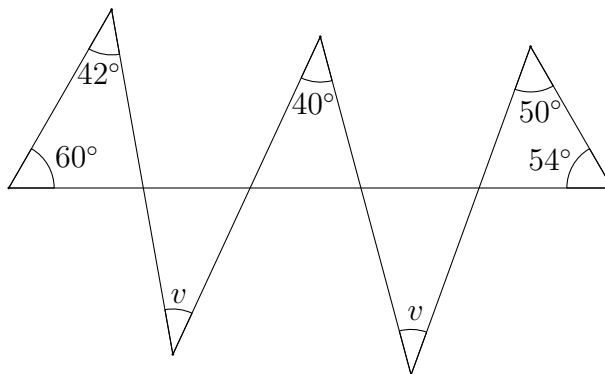
13. Søren, Torben and Ulrik each have 10 liters of water in a bowl. In turn, the boys do the following: The one whose turn it is pours one liter of water from his own bowl to each of the two others and then one liter of water onto the grass. Søren begins. At some point, Søren's bowl becomes empty. How many liters of water do Torben's and Ulrik's bowls totally contain at that point?
14. A teacher wishes to divide his class of n students into groups where each group has either 5 or 6 students. What is the largest value of n for which this is *not* possible?
15. A large clock has been broken, and only the minute hand is left. That hand can be rotated freely clockwise. If the hand is repeatedly rotated forward a certain number of minutes, it will sooner or later be back where it started. If, for example, one rotates it 45 minutes each time, it will be back at its starting position after that has been done four times. For how many of the numbers from 1 to 59 is it true that if the hand is rotated that number of minutes, it will be back at its starting position after five rotations?



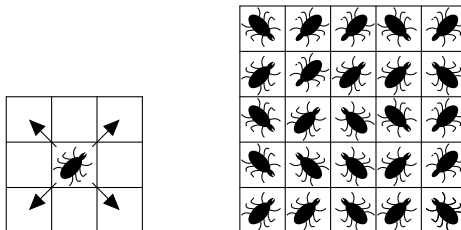
16. The figure shows three squares where the areas of the two smaller ones are 1 and 16, respectively. What is the area of the large square?



17. How many degrees is angle v in the figure?



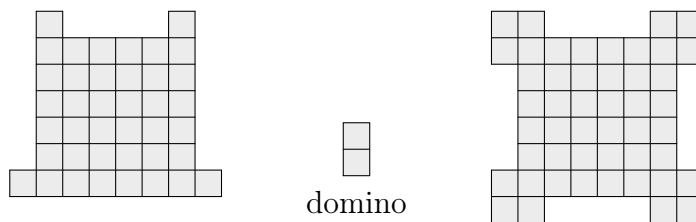
18. A flea is sitting in each of the 25 squares on a 5×5 board. Every second each of the fleas jumps to a square sharing exactly one corner with its square. There can be multiple fleas in one square. What is the maximal possible number of empty squares after one minute?



19. What is the value of the following expression?

$$(1^2 + 2^2 + 3^2 + \dots + 2021^2) - (0 \cdot 2 + 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + 2020 \cdot 2022)$$

20. The figure on the left can be covered with 20 dominoes in A ways, and the figure on the right can be covered with 24 dominoes in B ways. What is $\frac{B}{A}$?



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