

THE GEORG MOHR CONTEST 2020

First round

Tuesday, November 19 2019

Duration: 90 minutes

Aids allowed: none

Tick the answers on the included answer sheet

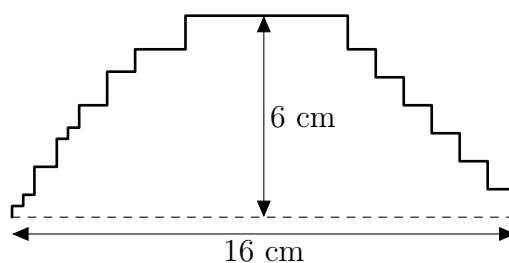
REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options, A, B, C, D and E.

One of these options is the correct answer.

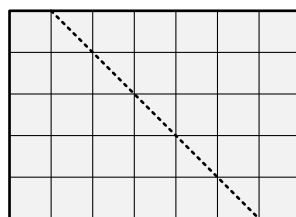
1. A piece of iron is bent at right angles so it forms a stairway across a model railway as shown in the figure. What is the length of the iron piece?



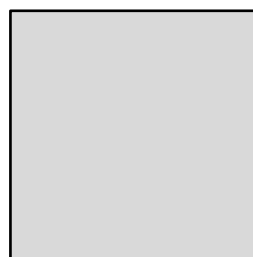
- A) 20 cm B) 22 cm C) 26 cm D) 28 cm E) it is not decidable

2. When the game board for *Hayjump* is folded along the dashed line it fits in the flat box. How much room is left in the box when the game board has been put down?

Game Board
Each square measures 10 cm × 10 cm

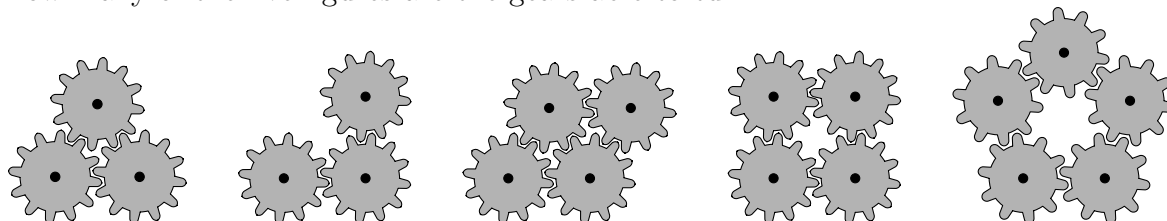


Æske
60 cm × 60 cm



- A) 180 cm² B) 1800 cm² C) 1750 cm² D) 1400 cm² E) 1350 cm²

3. In how many of the five figures are the gears able to turn?

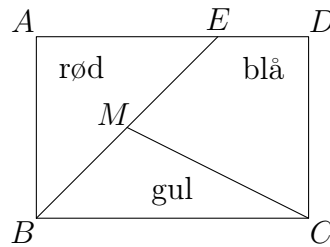


- A) 0 B) 1 C) 2 D) 4 E) 5

4. Alma and Bertha each have a 2 kroner and a 5 kroner coin. Alma starts by taking one of Bertha's two coins at random. Then Bertha takes one of Alma's three coins at random so that they again have two coins each. What is the probability that Alma now has two identical coins?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) $\frac{1}{4}$

5. The jigsaw puzzle in the figure consists of a red, a blue and a yellow piece, which together form the rectangle $ABCD$. The area of the rectangle is 24, and the area of the red piece ABE is 8. The point M is the midpoint of BE .



What is the area of the blue piece $CDEM$?

A) 6 B) 8 C) 9 D) 10 E) 12

6. Among the five people A , B , C , D and E , one always lies, while the remaining four always tells the truth. The five people make the following statements:

A : »Anyone who has a parrot lies.«

B : »I hate cauliflower.«

C : »I have a parrot.«

D : »The one who lies hates cauliflower.«

E : »Anyone who hates cauliflower has a parrot.«

Which of the five people lies?

A) A B) B C) C D) D E) E

7. How many of the integers $1, 2, 3, \dots, 100$ satisfy the equation

$$(x-1)(x-2)^2 + (x-1)^3(x-2)^4 + (x-1)^5(x-2)^6 = 0 ?$$

A) none B) 1 C) 2 D) 6 E) 11

8. How many of the integers $1, 2, 3, \dots, 5^{103}$ does 5^{100} divide?

A) 3 B) 4 C) 25 D) 103 E) 125

9. Which integer n satisfies that

$$n < \sqrt{42 + \sqrt{42 + \sqrt{42}}} < n + 1 ?$$

A) 5 B) 6 C) 14 D) 41 E) 42

10. In the expression $1 + 2 + 3 + \dots + n$, where n is a positive integer greater than 1, a number of parentheses are inserted, for example

$$1 + (2 + (3 + ((4 + 5))) + ((6 + 7) + (8 + 9) + 10).$$

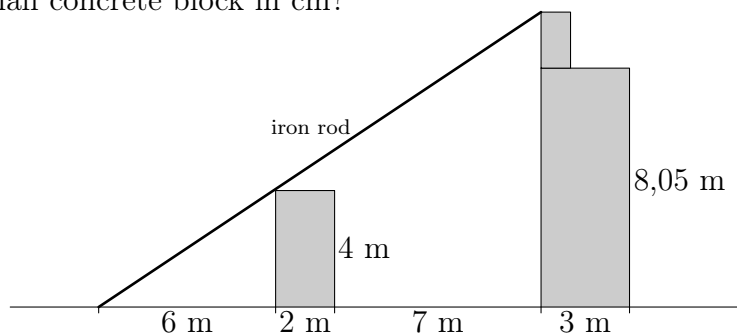
For clarity one wishes to colour the parentheses in such a way that each left parenthesis has the same colour as its corresponding right parenthesis, and such that two immediately adjacent parentheses, or parentheses only separated by a plus sign, have different colours. For example, the parentheses in the above example can be coloured using two colours. How many different colours may be needed in the worst case?

- A) 2 B) 3 C) 4 D) more than 4 E) the answer depends on n

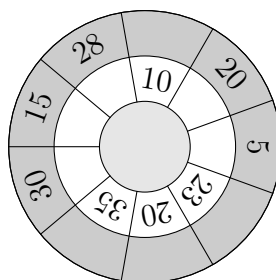
ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

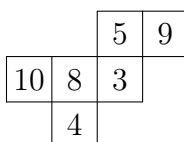
11. Amalie is twice as old as Benjamin. Benjamin is five years older than Clara. Clara is 12 years younger than Amalie. What is the sum of the childrens' ages in years?
12. There are three concrete blocks at a construction site. An iron rod rests on the medium-sized concrete block and just touches the corner of the small concrete block as shown in the figure. How tall is the small concrete block in cm?



13. The 19 regions in the figure each contain a number. Only some of the numbers are shown. The outer ring may be rotated to a position where the sum of a number in the outer ring and the number in the adjacent region in the inner ring is always the number in the middle region. In the position shown this is not the case. What is the number in the middle region?



14. The cardboard in the figure is folded into a cube. For each of the dice's eight corners, the three numbers on the adjacent three sides are multiplied, and the answer is written on a piece of paper. What is the largest of the eight numbers on the paper?

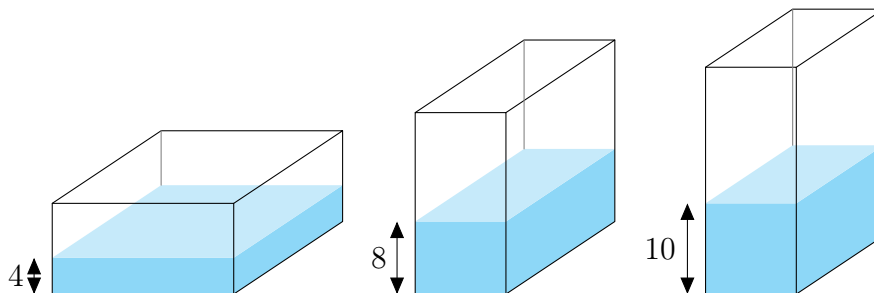


15. Omar writes all five digit numbers that consist of precisely one of each of the five digits 1, 2, 3, 4, and 5 on a list ordered by magnitude with the smallest first:

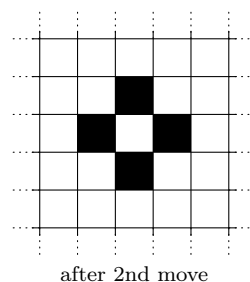
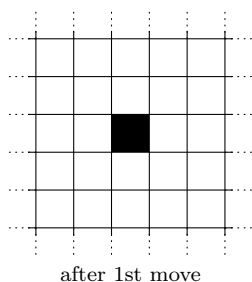
12345, 12354, \dots , 54321.

Then he splits the list in the middle. Which number is the first on the part of the list containing the largest numbers?

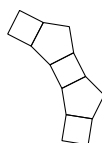
16. A glass box contains 2000 cm^3 water. The height of the water is either 4 cm, 8 cm or 10 cm depending on which side of the box is facing down. What is the volume (in cm^3) of the glass box?



17. For each two-digit number $10, 11, 12, \dots, 99$ Georg computes the first digit minus the last digit and writes the result on a piece of paper. What is the sum of all the results?
18. A computer game has an infinite squared game board where each square is initially white. At the first move, the computer colours one square black. In each of the following moves it simultaneously changes the colour of every square that has a neighbouring square of the opposite colour, i.e. all white squares with a black neighbouring square become black, and all black squares with a white neighbouring square become white. Two squares are neighbours if they have a common side. How many black squares are there after 1000 moves?



19. Some regular pentagons and squares with the same side length are put together alternately as shown in the figure. How many pentagons must be used in total to make a closed chain?



20. Agnes is told to write some positive integers on a blackboard. All the numbers must be distinct, and she must make sure that 3 divides exactly three of them, 5 divides exactly five of them, 7 divides exactly seven of them, and 19 divides exactly 19 of them. The largest of the numbers she writes she calls M . What is the smallest possible value of M ?

Sponsorer: Undervisningsministeriet, Jobindex, VILLUM FONDEN, William Demant Fonden, Georg Mohr Fonden og Matematiklærerforeningen.