## THE GEORG MOHR CONTEST 2019

First round

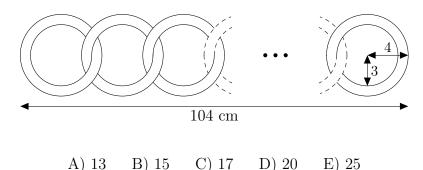
Tuesday, November 20 2018

Duration: 90 minutes Aids allowed: none Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

## MULTIPLE CHOICE PROBLEMS To each of the problems 1 – 10 there are five options, A, B, C, D and E. One of these options is the correct answer.

1. A number of identical cardboard rings are put together, forming a 104 cm long chain as shown. The outer radius of each cardboard ring is 4 cm, and the inner radius is 3 cm. How many rings does the chain consist of?



2. Seven sticks all have integer lengths, and their total length is 84. The two shortest sticks have a combined length of 17, while the two longest sticks have a combined length of 33. What is the largest number of sticks whose length may be an odd number?

A) 1 B) 2 C) 3 D) 4 E) 5

3. A disc with radius 5 is rolled along the line from A to B.



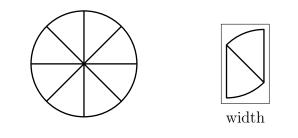
The figure shows the disc in both the start and the final positions. The distance from A to B is one of the following five numbers. Which one?

A)  $22\pi$  B)  $23\pi$  C)  $24\pi$  D)  $25\pi$  E)  $26\pi$ 

4. Ane chooses an integer. Benny chooses Ane's number plus 1, Charlotte chooses Benny's number plus 1, and Dorte chooses Charlotte's number plus 1. Now Else chooses Dorte's number multiplied by 4, and Freddy chooses Else's number plus 2. How many of the six chosen numbers does 4 divide?

A) 1 B) 2 C) 3 D) 4 E) it depends on the number chosen by Ane

- 5. From the numbers a, b, c and d, which are all greater than 2, one forms the numbers  $A = \frac{a+b}{c+d}, B = \frac{a\cdot b}{c+d}, C = \frac{a+b}{c\cdot d}$  and  $D = \frac{a\cdot b}{c\cdot d}$ . Which of these numbers is greatest?
  - A) A B) B C) C D) D E) it cannot be determined
- 6. A layer cake with a radius of 16 cm is cut into eight equally large pieces. Two of the pieces are put into a rectangular box as shown. What is the least possible width of the box?

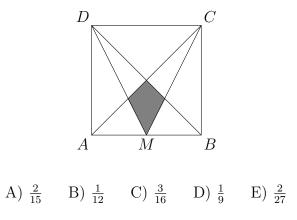


A) 8 cm B)  $2\pi$  cm C)  $8\sqrt{2}$  cm D)  $4\sqrt{3}$  cm E)  $4\pi$  cm

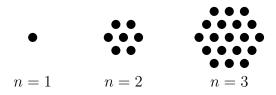
7. The numbers a, b and c satisfy a + b + c = 15, a + b - c = 5 and a - b + c = 25. What is the value of the expression a - b - c?

A) 15 B) 5 C) -5 D) -10 E) -45

8. In the square, M is the midpoint of the side AB. The side length of the square is 1. What is the area of the grey region?



9. The figure shows hexagons made of dots with n dots along each side. For n = 1 the figure consists of 1 dot, for n = 2 of 7 dots and so on.



How many dots are needed for the hexagon with n dots along each side?

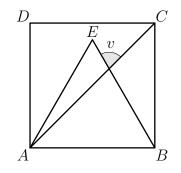
A) 
$$n^3 - 3n^2 + 8n - 5$$
  
B)  $6n - 5$   
C)  $n^2 + 3n - 3$   
D)  $3n^2 - 3n + 1$   
E)  $n^3 + 6n^2 - 7n + 1$ 

10. It is given that x = r is a solution to the equation  $x^3 + 1 = a \cdot x$ . Which of the following numbers is certainly a solution to the equation  $x^3 + 1 = a \cdot x^2$ ?

A)  $r^2$  B)  $r^3$  C)  $\sqrt{r}$  D)  $\frac{1}{r}$  E)  $a \cdot r$ 

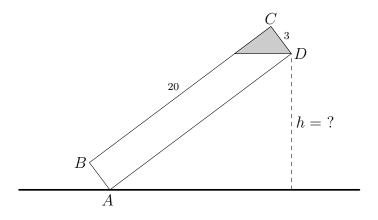
## ANSWER PROBLEMS The answer to each of the problems 11 - 20 is a positive integer

11. The figure shows a square ABCD and an equilateral triangle ABE.

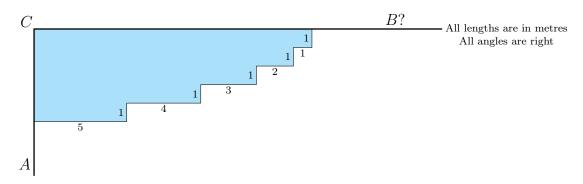


How many degrees is the marked angle v?

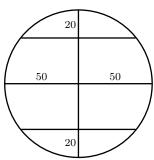
- 12. What is  $20192014 \cdot 20192015 20192012 \cdot 20192017$ ?
- 13. A rectangle *ABCD* with side lengths 3 and 20 has been tilted so much that the triangle cut off by a horizontal line through *D* constitutes one tenth of the rectangle's area. What is the vertical distance from the point *D* to the horizontal line at the bottom of the figure?



- 14. Victor has a part-time job where he every day has to pack a certain number of boxes and deliver them to the kiosk on the corner. Each box must contain a certain number of bags, each of which contains a certain number of small teddy bears. Victor has computed that he delivers 1155 teddy bears every day! One day he accidentally puts one extra bag in each box. As a result he happens to deliver 165 extra teddy bears that day. How many bags should there normally be in each box?
- 15. A pond has been constructed in one corner of a park. The figure shows the pond as seen from above with some lengths indicated. The landscape architect wishes to put up a low, straight fence connecting the point A with a point B. The point A lies 7 metres from C. How many metres from C must the point B be placed when one wishes that the triangle ABC contains the pond, and that the fence is as short as possible?



- 16. It takes Georg 30 minutes to peel potatoes for the family's supper. His mother can do it in 20 minutes. How many minutes does it take them if they help each other?
- 17. All positive integers dividing 505050 are written after one another in descending order. The first number is thus 505050, while the last two numbers are 2 and 1. What is the fourth number?
- 18. A circular window with a radius of 50 cm is equipped with bars as shown. Some of the lengths are given in the figure in cm. What is the total length (in cm) of the bars?



- 19. A jar contains red and green balls. If a ball is drawn at random, the probability that it is red is  $\frac{1}{6}$ . Now one ball is removed from the jar. If one then draws a ball at random, the probability of it being red is  $\frac{1}{7}$ . How many balls were in the jar initially?
- 20. Numbers have been placed in some of the spaces in a  $7 \times 7$  square. One must place numbers in the remaining spaces so that the sum of the nine numbers in each  $3 \times 3$  square is 2019. Which number must be put in the bottom right space?

10		8		11
7				
6				?