THE GEORG MOHR CONTEST 2018

First round

Tuesday, November 14 2017

Duration: 90 minutes
Aids allowed: none
Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options A, B, C, D and E. One of these options is the correct answer.

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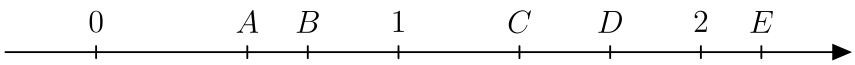
1. The numbers A, B, C, D and E are marked on the number line.



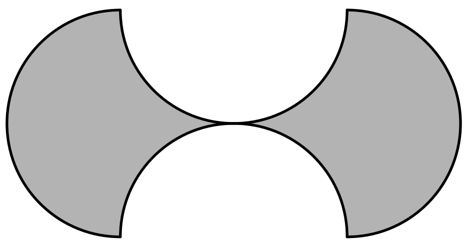
What is $A \cdot C$?

A) A B) B C) C D) D E) E

2. The numbers x and y are positive integers, and it is given that $3 \cdot x \cdot 4 \cdot y \cdot 20 = 4800$. What may with certainty be inferred from this?

A) $x = 2$ and $y = 10$ B) $x + y = 12$ C) either x or y equals 2
D) $x \cdot y = 20$ E) neither x nor y equals 4

3. The figure is composed of four half-circles, each with radius 5. What is the area of the figure?

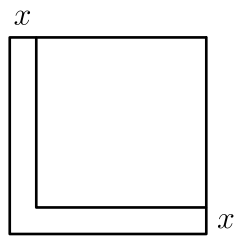


A) $25 \cdot \pi$ B) $25 \cdot \pi + \sqrt{5}$ C) 100 D) $25 + 25 \cdot \pi$ E) $50 \cdot \pi$

4. On different days, the three sisters Ida, Jane and Katrin walk from a cottage to a hilltop and back again. Ida walks with constant speed on the entire trip. Jane walks twice as fast as Ida on the way out, and half as fast as Ida on the way home. Katrin walks twice as fast as Jane on the way out and half as fast as Jane on the way home. Who did the round-trip fastest?

A) Ida B) Jane C) Katrin D) Jane and Katrin E) all three are equally fast

5. A square has area 4. The square is divided into two parts: a smaller square of area 3 and an L-shaped figure of area 1 and width x as shown. How big is the width x?

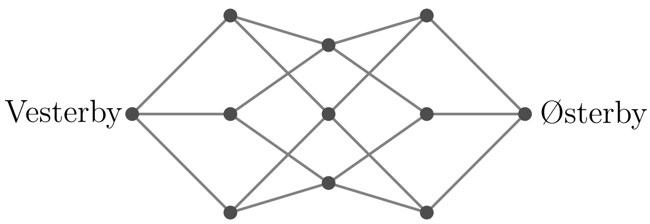


A) $\sqrt{3}-1$ B) $\sqrt{2}-1$ C) $\frac{1}{2}$ D) $2-\sqrt{2}$ E) $2-\sqrt{3}$

6. At a gathering of p people there are s liters of juice per person. The juice is poured into cans each containing m liters, and the cans are distributed on n tables. How many cans are there on each table on average?

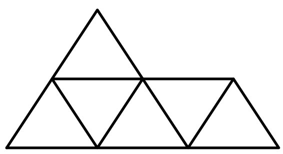
A) $\dfrac{s\cdot p}{m\cdot n}$ B) $\dfrac{p\cdot m}{n\cdot s}$ C) $\dfrac{m\cdot s}{p\cdot n}$ D) $\dfrac{p\cdot n}{s\cdot m}$ E) $\dfrac{n\cdot m}{p\cdot s}$

7. A car drives from Vesterby to Østerby along the shown grid. The trip consists of four segments each lasting one hour. Another car drives from Østerby to Vesterby, also along four segments each lasting one hour. The two cars start at the same time. What is the probability that they meet?



A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $(\frac{1}{3})^3$ D) $\frac{1}{9}$ E) $(\frac{1}{3})^2+(\frac{1}{2})^2$

8. Klara's math book contains a figure consisting of six triangles. Two of them must be painted blue, two of them red and the last two yellow. In how many ways can she do this?



A) 36 B) 3^6 C) 90 D) 120 E) 720

9. There are 12 presents numbered 1 through 12 on a table. Arne's favorite number is 2, Bent's favorite number is 3, Carl's favorite number is 4, Dora's favorite number is 5, and Ella's favorite number is 6. Arne, Bent, Carl, Dora and Ella walk up to the table in some order and receive the presents (from among those left) whose number their favorite number divides. It turns out that the last one gets two presents. Who is the last one?

A) Arne B) Bent C) Carl D) Dora E) Ella

10. For how many distinct positive integers n is the fraction

$$\frac{16n + 13}{3n + 2}$$

an integer?

A) 0 B) 1 C) 6 D) 29 E) infinitely many

ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

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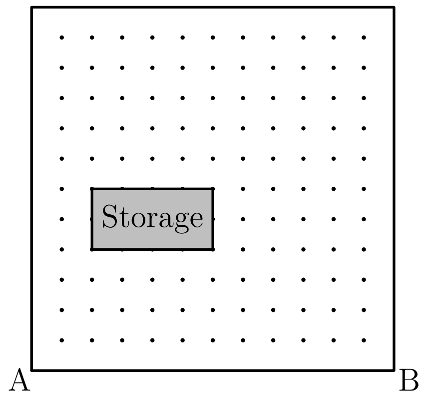
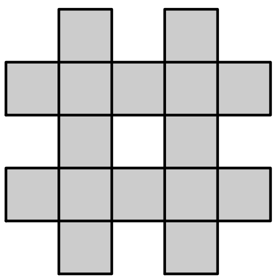
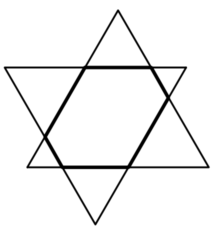
11. The figure depicts a show room measuring 12 m × 12 m seen from above. There are surveillance cameras in the corners A and B. Due to the shown storage room, whose walls extend from floor to ceiling, neither camera can watch the entire show room. How many square meters of the room are not visible from either of the cameras?



12. The numbers 1 to 16 are to be placed in the grey squares in the figure. Then one computes the sum S_1 of all the numbers in the two vertical bars and the sum S_2 of all the numbers in the two horizontal bars. What is the largest possible difference between the numbers S_1 and S_2?

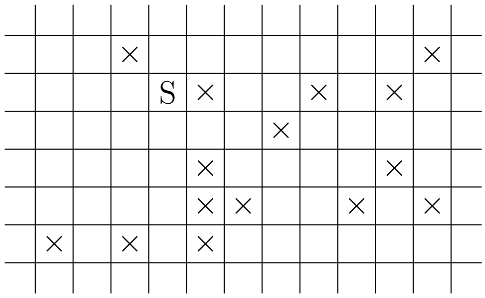


13. Two equilateral triangles with a circumference of 27 are placed so that their sides are parallel as shown in the figure. What is the circumference of the generated hexagon?



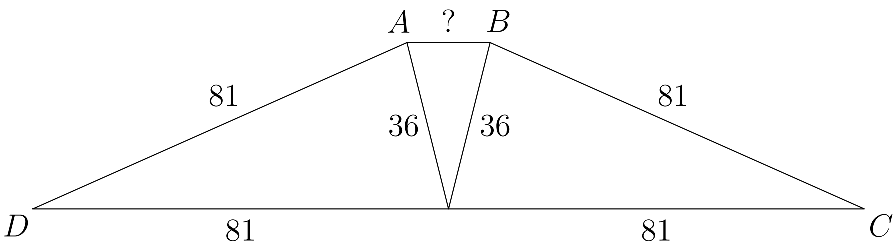
14. Seven girls and seven boys line up in a random order. In each round the teacher points at two adjacent children, who then switch places. How many rounds are at most needed to make the seven girls occupy the first seven places?

15. A game piece is placed on an infinitely large squared game board. In each move one is allowed to do one of four things: Move the piece 10 squares horizontally, 14 squares horizontally, 5 squares vertically or 7 squares vertically. How many of the squares marked with $\times$ is it possible to reach if the piece starts at the square S? One can use as many moves as one wishes.



16. 19 balls are numbered $2, 3, 4, \ldots, 20$. The balls must be painted in such a way that two balls have the same colour if the number on one ball divides the number on the other ball. If that is not the case, the balls may both have the same colour or different colours. How many different colours can one maximally use to paint the 19 balls?

17. The figure shows the quadrilateral $ABCD$ with several given lengths. What is the length of the segment AB?



18. Georg has placed 1001 buckets on the floor in one long row. In the first round, Georg puts a marble in each of the 1001 buckets. In the second round, he puts a marble in every second bucket, beginning with the first bucket. He continues like this for 1000 rounds where in round n he puts a marble in every n'th bucket, always starting by putting a marble in the first bucket. How many marbles does the last bucket contain at the end?

19. Determine the number

$$\left(1+\frac{1}{2}\right)^2\left(1+\frac{1}{3}\right)^2\left(1+\frac{1}{4}\right)^2\left(1+\frac{1}{5}\right)^2\cdots\left(1+\frac{1}{999}\right)^2.$$

20. For any number with at most three digits $n = abc$ one forms the number $T(n) = a\cdot 9 + b\cdot 3 + c$. For example $T(425) = 4\cdot 9 + 2\cdot 3 + 5 = 47$ and $T(T(425)) = T(47) = 0\cdot 9 + 4\cdot 3 + 7 = 19$. What is the largest possible value of $T(T(n))$?