THE GEORG MOHR CONTEST 2016

First round

Tuesday, November 10 2015

Duration: 90 minutes
Aids allowed: none
Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options A, B, C, D, and E. One of these options is the correct answer.

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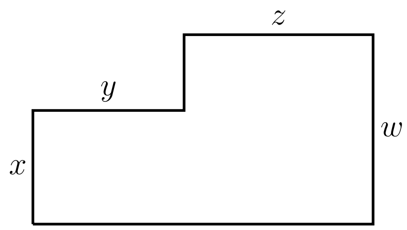
1. The figure shows a building site, with some of the side lengths given. All angles on the figure are 90°. What is the circumference of the building site?



A) $x + y + z + w$ B) $2y + 2w + 2z$ C) $2x + y + z + w$
D) $x + y + z + w + y + z$ E) $xy + zw$

2. Georg thinks of an integer. First he multiplies the number by 5 and subtracts 7 from the result. Then he multiplies the new result by 9 and obtains the number x. What can one say about x with certainty?

A) x is odd B) x is even C) 3 divides x D) 5 divides x E) 7 divides x

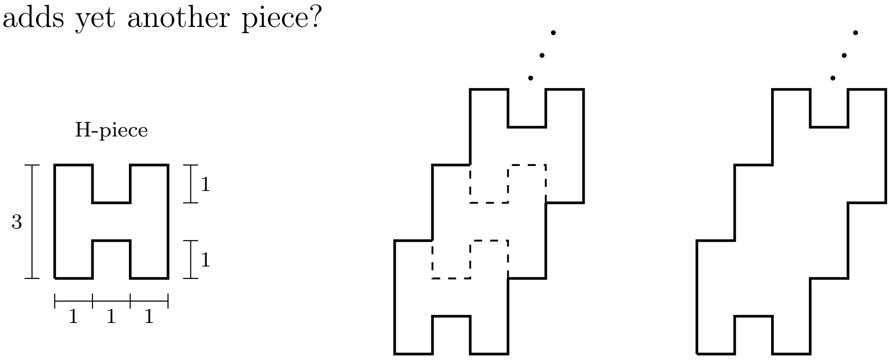
3. Each second, a slot machine chooses one of the symbols ♡, ♠, ♢ and ♣ at random. What is the probability that the first four symbols it chooses are identical?

A) $4 \cdot \frac{1}{4}$ B) $\frac{1}{4 \cdot 4}$ C) $\frac{1}{4 \cdot 3 \cdot 2 \cdot 1}$ D) $\frac{1}{4^4}$ E) $\left(\frac{1}{4}\right)^3$

4. It is known that the number $x = 4$ is a solution to this equation: $a + b \cdot \sqrt{x^3 + 36} = c \cdot x$. What can be deduced concerning the equation $a + b \cdot \sqrt{x^6 + 36} = c \cdot x^2$?

A) $x = 2$ is a solution B) $x = 4$ is a solution C) $x = 8$ is a solution
D) $x = 16$ is a solution E) none of these

5. By combining H-shaped pieces in the shown pattern one can build figures. As more H-pieces are added, the outer circumference of the figure increases. A figure consists of n pieces, where n is at least 1. By how much does the outer circumference increase when one adds yet another piece?



A) 5 B) 6 C) 11 D) 16 E) it depends on n

6. What is the last digit in the number $2 + 2^2 + 2^3 + 2^4 + \cdots + 2^{100}$?

A) 0 B) 2 C) 4 D) 6 E) 8

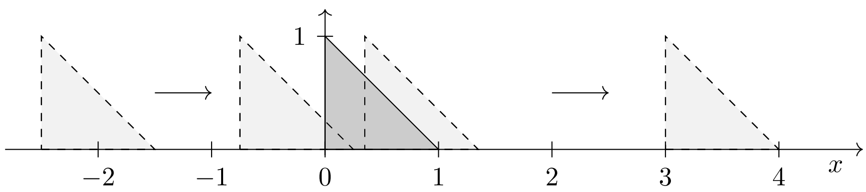
7. Integers a, b and c are known to satisfy $a^2b^3c^4 = -27$. How many possibilites are there for the value of c?

A) 0 B) 1 C) 2 D) 3 E) 4

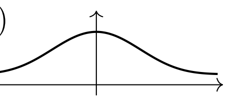
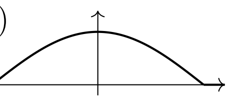
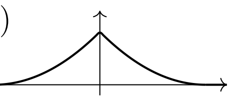
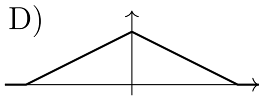
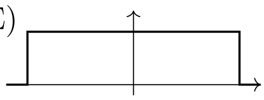
8. Which of the following expressions is $\frac{x+1}{1-x^2} - 1$ equal to?

A) $\frac{x}{1-x}$ B) $-x-1$ C) $\frac{x}{1-x^2}$ D) $-\frac{1}{x}$ E) $\frac{x-x^2}{1-x^2}$

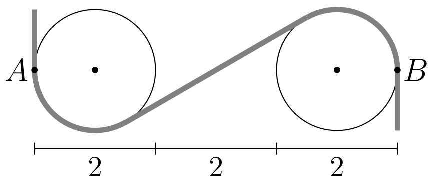
9. A triangle lies in a coordinate system as shown. Another triangle of the same shape slides along the x-axis with constant speed and passes over it.



The area of the two triangles' overlap is registered in a coordinate system with the time on the x-axis. It starts out at 0 and attains its maximal value when the two triangles exactly cover each other. Which of the graphs below is correct?

A) 
B) 
C) 
D) 
E) 

10. A rope is pulled tight around two sheaves as shown in the figure. The rope meets the first sheave at point A and leaves the second sheave at point B. The sheaves have radius 1 and a mutual distance of 2. The points A, B and the centres of the sheaves are collinear.



How long is the piece of robe between A and B?

A) $2\sqrt{3}+\frac{4}{3}\pi$ B) $2\sqrt{5}+\pi$ C) $\frac{3}{2}\pi+3$ D) $3+\sqrt{20}$ E) 8

ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

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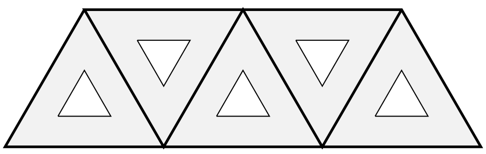
11. Given any number, one may perform one of the operations below and thus get a new number. Begin with the number 15. Then perform each of the four operations exactly once in any order you wish. What is the largest possible end result?

- square the number
- subtract 3 from the number
- append the digit 0 to the number
- add 5 to the number

12. The figure shows a wall built from five identical plasterboards, each shaped like an equilateral triangle. In the middle of each board, a window, also shaped like an equilateral triangle, has been sawed out. The windows' edge lengths are one third of the boards' edge lengths. Each window has an area of 1 m^2.

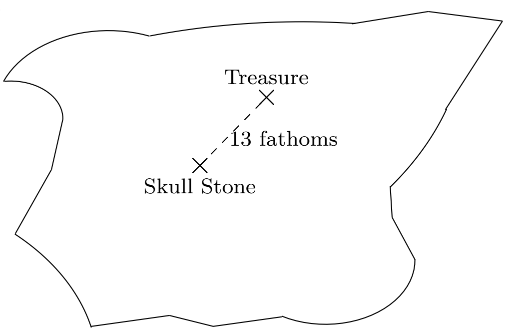


What is the combined area (in m^2) of the wall when the windows are excluded?

13. An old chart contains this text:

> *Walk from the Skull Stone 18 fathoms East, 4 fathoms South, X fathoms West, 17 fathoms North, 3 fathoms East, 8 fathoms South and finally 6 fathoms West, then you are standing exactly 13 fathoms from the Skull Stone, and here the treasure is buried. Guess the number X, which is not 3, and you may find the treasure.*

What is the number X?



14. 100 points are marked on a circle. How many segments is it possible to put between two of the points without any of them intersecting? Sharing a common endpoint does not count as intersecting.

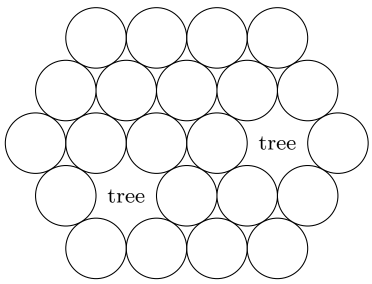
15. A positive integer is called a *palindrome* if it is identical to its mirror image. For example, 57475 is a palindrome, while 1227 is not. You are given that x is a four-digit palindrome, and that $x + 852$ is a 5-digit palindrome. What is x?

16. Bertha is playing a one-man game. At the beginning, there are $1, 2, 3, \ldots, 999$ or 1000 marbles in a bowl. Bertha wins if she can remove all the marbles from the bowl by using these two moves as many times as she likes:

    - remove five marbles
    - remove half the marbles, provided there is an even number

    For how many of the starting values $1, 2, 3, \ldots, 999, 1000$ is it possible for Bertha to win?

17. A market place with two old trees must be tiled with 22 big round tiles colored blue, gray and green. The tiles must be placed as shown in the figure. In how many ways can the tiles be chosen when two adjacent tiles cannot have the same color?



18. You are given the the two numbers x and y satisfy

$$4x \leq 3y + 2016 \leq 2x + 2016.$$

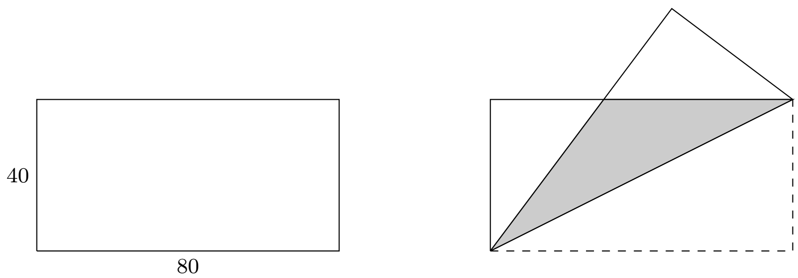
    What is the largest possible value of y?

19. The numbers a, b, c, d, e and f are positive integers below 1000, and they satisfy

$$4a = 5b = 6c = 7d = 8e = 9f\ .$$

    What is the number a?

20. A piece of cardboard measuring 40×80 is folded along the diagonal as show.



    What is the area of the overlap?