THE GEORG MOHR CONTEST 2015
First round
Tuesday, November 11, 2014

Duration: 90 minutes
Aids allowed: none
*Tick the answers on the included answer sheet*

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS
To each of the problems 1 − 10 there are five options A, B, C, D, and E.
**One of these options is the correct answer.**

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1. The dashed line AB in the figure has length 30. In addition, a broken straight line l has been drawn from A to B. Together with the line AB, the broken line forms six squares. What is the length of the broken line l?
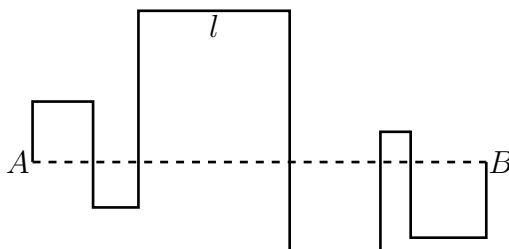
A) 60   B) $30 + 30\sqrt{2}$ C) 90   D) $60 + 30\sqrt{2}$ E) 120

2. In the figure below, the number 2 has been placed in the middle circle. The remaining numbers from 1 to 9 must be placed in the other circles in such a way that the arrows point from a smaller to a larger number.
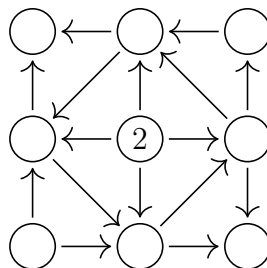
In how many ways can this be done?

A) 1   B) 2   C) 4   D) 8   E) it is not possible

3. The price of pineapples varies from shop to shop, see below. Rikke needs ten pineapples. Which of the shops A, B, C, D, E should she choose to pay the least amount?

   A) The unit price at A is 12 kr.

   B) The unit price at B is 17 kr., and one can buy a tray with three for 35 kr.

   C) The unit price at C is 13 kr., and buying seven gives the eighth for free.

   D) The unit price at D is 14 kr, and today one gets a 10 % discount on the combined purchase.

   E) The unit price at E is 20 kr., and a bag with four is sold for 38 kr.

4. The three integers a, b and c satisfy $a + b - 1 = b + c = c + a + 3$. Which of the numbers a, b and c is greatest?

   A) a B) b C) c D) they are all equal   E) it cannot be decided

5. Anna, Berit, Cecilie and Dorte each either always tell the truth or always lie. The girls state the following:

   Anna: "At least two among us lie."
   Berit: "I am the only one among us who lies."
   Cecilie: "I am the only one among us who tells the truth."
   Dorte: "We all tell the truth.."

   How many of the four girls lie?

   A) 0   B) 1   C) 2   D) 3   E) 4

6. A jewellery box contains twice as many white pearls as blue pearls. A blue pearl can be exchanged for nine white pearls. If all the blue pearls are exchanged for white pearls there will be P white pearls in the box. What can one say for certain about the number P?

   A) P is an even number   B) P is an odd number   C) 9 divides P
   D) 11 divides P E) none of these can be deduced

7. A clock face with a radius of 1 meter is placed on top of a clock face with a radius of 3 meters. Now the smaller clock face is rolled along the larger clock face. Which number on the smaller clock face is on top when it is adjacent to 9 o'clock on the larger clock face?
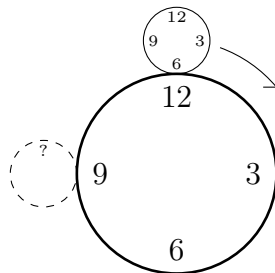

   A) 3   B) 6   C) 9   D) 12   E) none of these numbers are on top

8. Each cell in the table below is to be filled with integers in such a way that the sum of the numbers in any four adjacent cells is always 48. Three cells have already been filled. Which number must be put in the cell marked with an x?

1			17					x					11

   A) 12   B) 15   C) 17   D) 19   E) it cannot be decided

9. A square is cut into eight identical right-angled triangles as shown. The triangles can be assembled to either one, two, three or four squares. Around each square a circle passing through all four corners is drawn.
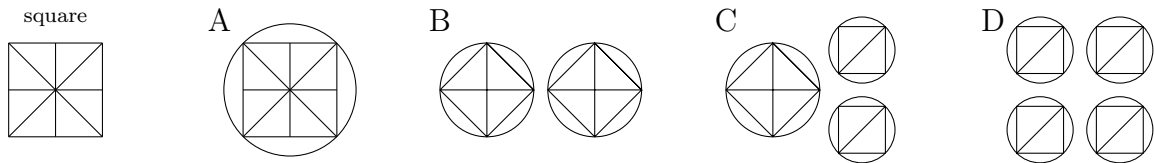
In which of the situations does the circles have the largest combined area?

A) A   B) B   C) C   D) D   E) the areas are equal

10. A box without a lid is to be painted outside and inside. Each of the five outer faces and five inner faces are painted either red or blue, and it is completely random which colour is chosen for each face.
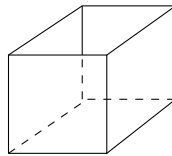

What is the probability that at least one face gets the same colour on the outside as on the inside?

A) $\frac{1}{2}$ B) $\frac{9}{10}$ C) $(\frac{1}{2})^5$ D) $\frac{31}{32}$ E) $\frac{4}{5}$

ANSWER PROBLEMS The answer to each of the problems 11 − 20 is a positive integer

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11. All of Julie's cousins live on either Lolland, Langeland or Læsø. She says that 15 of her cousins do not live on Lolland, that 17 do not live on Langeland, and that 18 do not live on Læsø. How many cousins does Julie have?

12. What is the smallest integer consisting of only the digits 1, 2 and 3, which contains each of the three digits at least once, and which 3 does not divide?

13. What is the value of the number $\dfrac{3^{2015} + 3^{2014}}{3^{2014} - 3^{2013}}$?

14. Figure A shows a construction made of iron rods, constituting the frame of a box with a square base. The combined length of the iron rods is 48 m. If the skeleton was reinforced with four additional vertical rods as shown in Figure B, the combined length would be 56 m.


figure A   figure B
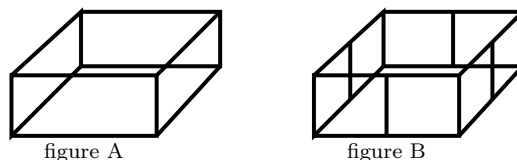
What is the volume of the box (in m³)?

15. The figure shows a hexagon in which all angles are equal. A line intersects one of the sides at an angle of 102° as shown. How large is the angle marked with v (in degrees)?
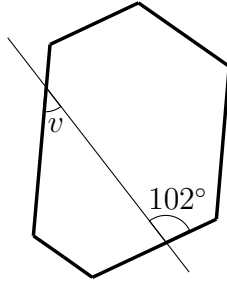
16. A giant abstract wall-painting is painted on a rectangular canvas which is 4 meters tall and 18 meters wide. A few lines have been drawn on the canvas, and furthermore three of the regions have been painted gray. What is the combined area of the three gray areas (in m^2)?
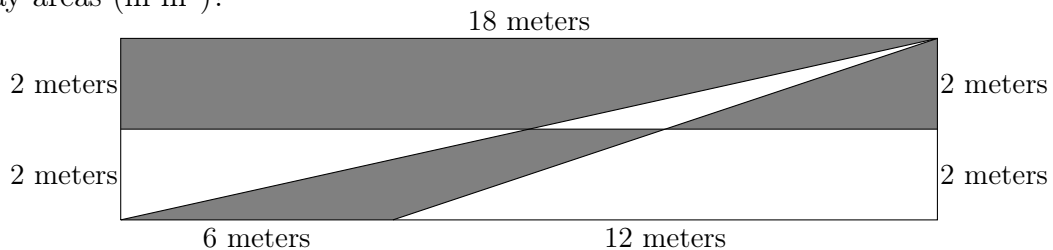
17. Ole travels from cell A to cell B. From a cell he can proceed to a neighbouring cell by one horizontal step or one vertical step. He gets 1 point for each horizontal step (right or left) and 2 points for each vertical step (up or down). Each cell must be visited at most once.
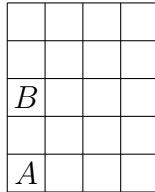
How many points can he obtain at most?

18. The box shown below has length 6, and the end is square with a side length of 2. A string is to be wrapped around the box from A to B via the front, the lid, the back and the bottom.
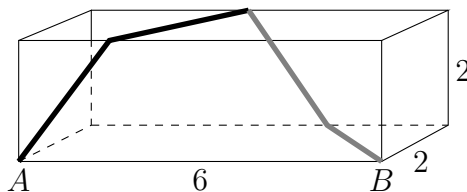
What is the shortest possible length of such a string?

19. A box contains sixty notes numbered from 1 to 60. Another box also contains sixty notes numbered from 1 to 60. Sixty people each draw a note at random from each box and multiply their two numbers. If the result is divisible by 6, the person gets a soft drink. How many soft drinks will at most be needed?

20. Ten friends went on a holiday together. Every day at most three of them did not use the hotel's swimming pool, and none of them used the swimming pool the same number of days as any of the others. This would not have been possible if the holiday had lasted one day less. How many days did the holiday last?