

THE GEORG MOHR CONTEST 2014

First round

Tuesday, November 12, 2013

Duration: 90 minutes

Aids allowed: none

Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

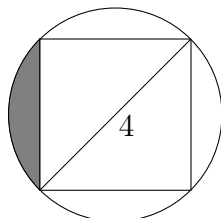
To each of the problems 1 – 10 there are five options A, B, C, D, and E.
One of these options is the correct answer.

1. Omar has written his name on a glass plate: OMAR. If one turns the plate and/or looks at it from the other side, the inscription looks different. How many of the following possibilities can be obtained?

ROMA ROMA RAMO RAMO
OMAR OMAR OVAR ROMA

A) 2 B) 3 C) 4 D) 5 E) 6

2. A square in which the diagonal has length 4 is inscribed in a circle as shown. What is the area of the gray region?



A) $2\pi - 2$ B) $\pi - 2$ C) $\pi^2 - 8$ D) $\pi + 2$ E) $4\pi - 2$

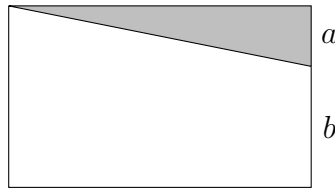
3. The numbers from 1 to 9 are to be distributed in a 3×3 grid. The three horizontal rows must each have an odd sum, and the three vertical columns must each have an even sum. Some of the numbers are already filled in:

		2
4		
		9

In how many ways can the rest of the grid be filled?

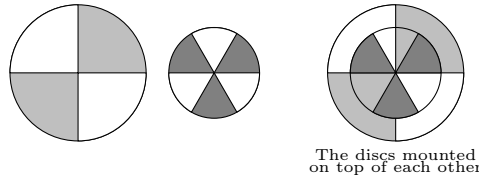
A) 1 B) 2 C) 48 D) 96 E) it is impossible

4. A rectangle is divided in two parts as shown in the figure. The area of the white part is five times as large as the area of the gray part. Determine the fraction $\frac{a}{b}$, where a and b are the lengths indicated in the figure.



- A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{5}$ E) $\frac{1}{2}$

5. The figure shows a large and a small glass disc. Each glass disc is divided into equal sections, which are alternately painted gray and transparent. The small disc is mounted on top of the large disc. The two discs may rotate independently, and when they are rotated, it alters which parts of the support are visible. The figure shows the discs in their initial position. How large an angle must the small disc be rotated counter-clockwise in order to make as much as possible of the support visible?

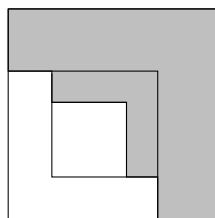


- A) 0° B) 30° C) 45° D) 60° E) one can see equally much of the support however the discs are rotated with respect to each other

6. Which of the following numbers is smallest?

- A) $\frac{3}{103}$ B) $\frac{1}{33}$ C) $\frac{301}{1001}$ D) $\frac{3}{100}$ E) $\frac{30}{1003}$

7. Four squares are placed inside each other as shown. Each new square has an area that is half as large as the preceding square. In the first and third squares, the part which is not covered by the following square is coloured. How large a fraction of the area of the large square does the coloured domain make up, totally?



- A) $\frac{5}{8}$ B) $\frac{3}{4}$ C) $\frac{13}{16}$ D) $\frac{1}{2}$ E) $\frac{9}{16}$

8. Peter has written two positive integers x and y on a piece of paper which he shows to his friends A, B, C, D, and E. The five friends state the following:

- A) $x + y$ is odd B) 3 divides y C) $y = 2x$
D) $x \cdot y$ is odd E) $x < y$

Peter tells us that exactly one of the five friends lies. Who?

9. A man has his entire fortune distributed in solid fur bags, each containing 49 gold coins. One day he redistributes the fortune, so that it lies instead in 35 boxes with the same number of coins in each box. There are various rumours telling how many gold coins his fortune consists of. Which of the five rumours A, B, C, D, and E below may be true?

A) 70705 B) 6335 C) 2450 D) 70707 E) 4931

10. You are given that exactly one of the numbers 14, 15, 16, 20, 25 is a solution to the equation

$$5x^5 + 3x^3 + x = 3604065 + 4x^4 + 2x^2.$$

Which of the numbers is it?

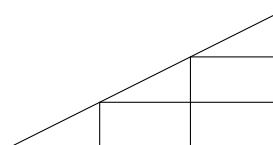
A) 14 B) 15 C) 16 D) 20 E) 25

ANSWER PROBLEMS The answer to each of the problems 11 – 20 is a positive integer

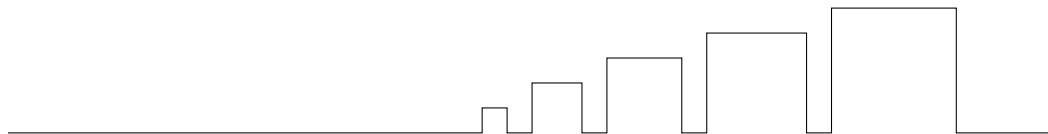
11. George has a rectangular piece of paper, the short edge 40 cm and the long edge 56 cm. He folds it along the line connecting the midpoints of the two longest edges (see figure). The paper then forms a new smaller rectangle, which he folds in the same manner, etc. What is the circumference, in cm, of the rectangle George gets after having folded the paper six times in this way?



12. Elly is twice as old as Anni. Dave is four years older than Cille. Anni is three times as old as Beth. Elly is 72 years old. Together, Dave and Cille are half as old as Anni. How many years old are Cille and Beth together?
13. Each employee in the engineering company *Black & Red* has his own individual logo consisting of a right-angled triangle divided into six regions as shown. Each region is either black or red, and it is allowed that all six regions have the same color. What is the maximum number of employees in the company?

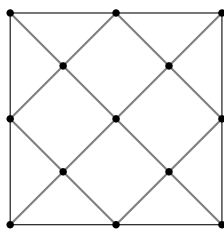


14. The figure shows five houses standing next to each other with a spacing of 10 meters. The first house is 10 meters tall and 10 meters wide, the second one is 20 meters tall and 20 meters wide, the third one is 30 meters tall and 30 meters wide, etc.



What distance, in meters, must a person be standing to the left of the smallest house in order to be just able to see the top of the fifth house? (We disregard the person's height.)

15. The numbers from 1 to n are shown on a long list. From the list, all numbers divisible by 3 and all numbers divisible by 5 are now removed. Afterwards, 240 numbers remain on the list. You are given that both 3 and 5 divide the number n . What is n ?
16. The gilt on the lattice gate shown below needs refreshing. Unfortunately, there is not enough gold paint! After the outer frame has been gilded, it thus turns out that half of the paint has been used. For how many of the 16 tilted pieces is there enough paint, at most?



17. When one joins the food club PITAKURS, one is given a club name. A club name only consists of the three vowels I, A, and U and the five consonants P, T, K, R, and S. All club names consist of five letters, and there are never more than two consonants next to each other. The words STIKA and TTUKK are thus allowed names, while SPRAT cannot be used. How many club names are possible with T as the second letter and K as the fourth one?
18. Ole mentions the numbers 0, 3, 6, 9, ... while Hans counts backwards from 200. Thus, Hans says 200 in the moment that Ole says 0, he says 199 when Ole says 3 etc. Every time Ole and Hans have said a number, Peter multiplies their two numbers and writes the result on a blackboard. What is the largest number which will occur on the blackboard?
19. Determine the least number n such that

$$x^{10} - 1001x^7 + 1 > 0$$

for all $x \geq n$.

20. Mette has forgotten her four-digit pin code! Fortunately, she remembers that when the two-digit number formed by the last two digits is subtracted from the two-digit number formed by the first two digits the result is 9. Furthermore she remembers that the sum of the squares of the first and last digits minus the sum of the squares of the second and third digits is exactly the two-digit number formed by the last two digits. What is Mette's four-digit pin code?

(By the *square* of a number is meant the number to the second power.)