THE GEORG MOHR CONTEST 2013

First round

Tuesday, November 13 2012

Duration: 90 minutes
Aids permitted: none
Answer by ticking the enclosed answering sheet

MULTIPLE CHOICE PROBLEMS

Each of the problems 1–10 has five possible answers, A, B, C, D, and E. One of these is correct.

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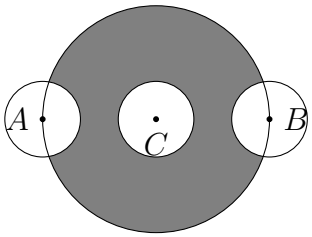
1. There are five boxes in Ole's room. Each box contains five smaller boxes, each of which in turn contains five even smaller boxes. How many boxes are there altogether?

   A) $5+5+5$ B) 3^5 C) $5\cdot 4\cdot 3$ D) $5+5^2+5^3$ E) $(5\cdot 3)\cdot(5\cdot 2)\cdot 5$

2. The dots on a die have been replaced by the letters M, R, S, S, U and U. Marie and Hans each throw the die three times. Which of the following outcomes is most likely?

   A) SUR MUR B) SUR MUS C) MUR SUM
   D) URS RUS E) MUS SUS

3. The big circle has radius 3 and centre C. The small circles have radius 1 and centres A, B, and C. The points A and B lie on the circumference of the big circle. What is the area of the shaded region?



   A) less than 4π B) 4π C) between 4π and 7π D) 7π E) more than 7π

4. The diagram below must be completed in such a way that each row and each column contain the four symbols $\times$, $\circ$, Δ and $*$. Some of the squares are already filled in. In how many different ways can the rest of the diagram be completed?

$\circ$			
$*$		Δ	
		$\times$	

   A) 1 B) 2 C) 3 D) more than 3 E) it is not possible

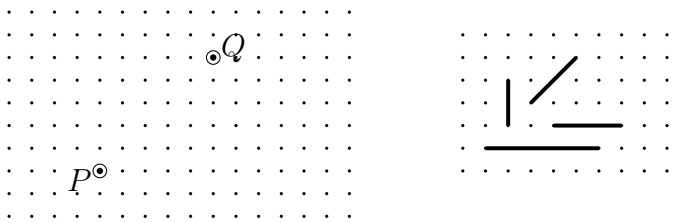
5. A display shows the four digits 1, 2, 5, and 7. Every second, one of the three leftmost digits is randomly chosen and moved all the way to the right. For instance

$$\ldots \to 1527 \to 1275 \to 2751 \to 2715 \to \ldots$$

If the display at a given moment shows 1527, what is the probability that it shows 1527 again two seconds later?

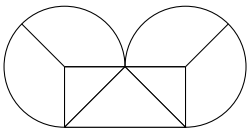
A) $\frac{1}{4}$ B) $\frac{1}{6}$ C) $\frac{1}{9}$ D) $\frac{2}{5}$ E) $\frac{1}{24}$

6. Hans must draw a route starting at P and ending at Q by placing segments with length and direction as those shown one after another. It is allowed to use the same type of segment as many times as one wishes. The route may go outside the shown region, and it may cross itself. What is the smallest number of segments one can use to draw such a route?



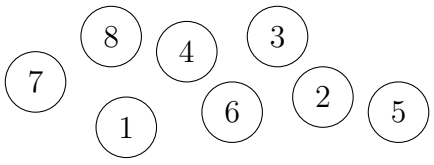
A) 5 B) 6 C) 11 D) more than 11
E) it is not possible to draw such a route

7. What is the combined length of all circular arcs and line segments in the figure below? The circles have radius 1.



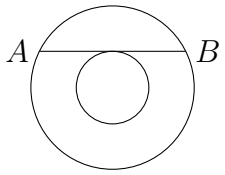
A) $8 + 2\sqrt{2} + 3\pi$ B) $10 + \frac{3}{2}\pi$ C) $6 + 4\sqrt{2} + \frac{3}{2}\pi$
D) $8 + 2\sqrt{2} + \frac{3}{2}\pi^2$ E) $10 + 3\pi$

8. Peter jumps from stone to stone. He starts on stone 1, ends on stone 8, and lands exactly once on each of the other stones. Each jump may be as short or long as desired. In how many different ways can he plan a jumping route, provided he wants the fourth jump to take him to stone 7?



A) 5^6 B) 15 C) 120 D) 224 E) 7^5

9. A big and a small circle have the same centre. The chord AB is tangent to the small circle and has length 16. What is the area of the ring between the circles?



A) 100 B) 32π C) 64π D) 256π
E) it is undecidable from the given information

10. a, b, c, and d are integers. From exactly one of the equations A, B, C, D, E, one can deduce that 14 divides $a \cdot b$. Which one?

A) $7a + 8b = 14c + 28d$ B) $14a + 28b = 7c + 8d$
C) $14a + 8b = 7c + 28d$ D) $7a + 28b = 14c + 8d$
E) $28a + 8b = 14c + 7d$

ANSWER PROBLEMS

The answer to each of the problems 11–20 is a positive integer.

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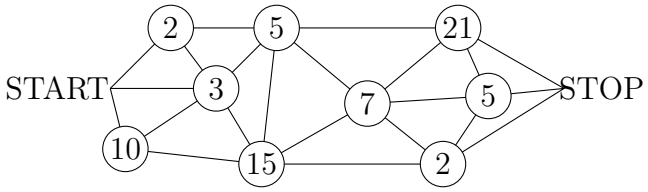
11. A number of bicycles, tricycles and four-wheeled strollers are parked in the courtyard. There are twice as many strollers as tricycles, and there are three times as many bicycles as strollers. Altogether, there are 184 wheels. How many tricycles are there?

12. A rectangle is twice as long as it is wide. Its diagonal has length $\sqrt{45}$. What is the area of the rectangle?

13. In the number maze you enter at START, follow the paths and exit at STOP. Notice the numbers you pass on the way. The goal is to choose a route through the maze making the product of all the numbers encountered equal to 210. How many routes satisfy this requirement?

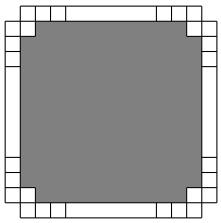


14. Numbers a and b are substituted into the expression

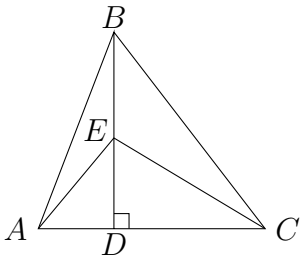
$$\frac{a^2 - b^2}{a - b}$$

with $a > b$. The numbers a and b must be chosen among 1, 2, ..., 10. How many different results can be obtained?

15. The figure shows a flower bed with a paved path surrounding it. The tiles measure 1 meter $\times$ 1 meter, and a total of 104 tiles have been used. The draft shows the tiling at the corners correctly, while the tiling at the sides, which are all the same length, is shown incompletely. What is the area of the flower bed in square meters?



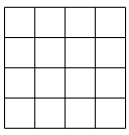
16. In the sketch, BD is an altitude in triangle ABC, and E is the midpoint of BD. It is given that $|AC| = 13$, $|BE| = 6$ and $|EC| = 10$. What is the area of triangle ABE.



17. What is the smallest positive integer n with the property that the number $180 \cdot n$ is a perfect cube? (A perfect cube is a number expressable as m^3, where m is a positive integer.)

18. A big bag contains red, yellow, green, blue and black socks mixed together, no less than 50 of each color. Peter draws socks from the bag without looking. What is the least number of socks he needs to pick up to be sure that he can create at least ten matching pairs?

19. The numbers from 1 to 16 are written with invisible ink in the squares below. The sum of the four numbers in the leftmost vertical column is 19. The sum of the numbers in the topmost row is 58. Each of the numbers in the third row is exactly twice the number immediately above it. What is the sum of the numbers in the bottom row?



20. A positive integer n has the property that there exists exactly one positive integer k such that

$$54n < 55k < 56n \quad .$$

What is the greatest possible value of n?