Thursday, July 8, 2010

Problem 4. Let P be a point inside the triangle ABC. The lines AP, BP and CP intersect the circumcircle Γ of triangle ABC again at the points K, L and M respectively. The tangent to Γ at C intersects the line AB at S. Suppose that $SC = SP$. Prove that $MK = ML$.

Problem 5. In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:

- *Type* 1: Choose a nonempty box B_j with $1 \leq j \leq 5$. Remove one coin from B_j and add two coins to B_{j+1}.
- *Type* 2: Choose a nonempty box B_k with $1 \leq k \leq 4$. Remove one coin from B_k and exchange the contents of boxes B_{k+1} and B_{k+2}.

Determine whether there is a sequence of such operations that results in boxes B_1, B_2, B_3, B_4, B_5 being empty and box B_6 containing exactly $2010^{2010^{2010}}$ coins. (Note that $a^{b^c} = a^{(b^c)}$.)

Problem 6. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers. Suppose that for some positive integer s, we have

$$a_n = \max\{a_k + a_{n-k} \mid 1 \leq k \leq n-1\}$$

for all $n > s$. Prove that there exist positive integers ℓ and N, with $\ell \leq s$ and such that $a_n = a_\ell + a_{n-\ell}$ for all $n \geq N$.

Language: English

Time: 4 hours and 30 minutes
Each problem is worth 7 points