Language: **English** Day: **1**

# 49th INTERNATIONAL MATHEMATICAL OLYMPIAD
## MADRID (SPAIN), JULY 10-22, 2008

*Wednesday, July 16, 2008*

**Problem 1.** An acute-angled triangle $ABC$ has orthocentre $H$. The circle passing through $H$ with centre the midpoint of $BC$ intersects the line $BC$ at $A_1$ and $A_2$. Similarly, the circle passing through $H$ with centre the midpoint of $CA$ intersects the line $CA$ at $B_1$ and $B_2$, and the circle passing through $H$ with centre the midpoint of $AB$ intersects the line $AB$ at $C_1$ and $C_2$. Show that $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, $C_2$ lie on a circle.

**Problem 2.** (a) Prove that

$$\frac{x^2}{(x-1)^2}+\frac{y^2}{(y-1)^2}+\frac{z^2}{(z-1)^2}\geq 1$$

for all real numbers $x$, $y$, $z$, each different from 1, and satisfying $xyz=1$.

(b) Prove that equality holds above for infinitely many triples of rational numbers $x$, $y$, $z$, each different from 1, and satisfying $xyz=1$.

**Problem 3.** Prove that there exist infinitely many positive integers $n$ such that $n^2+1$ has a prime divisor which is greater than $2n+\sqrt{2n}$.

*Language: English*

*Time: 4 hours and 30 minutes*
*Each problem is worth 7 points*