The Georg Mohr Contest 2022
Second Round

Tuesday, 11 January 2022 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. The figure shows a glass prism which is partially filled with liquid. The surface of the prism consists of two isosceles right triangles, two squares with side length 10 cm and a rectangle. The prism can lie in three different ways. If the prism lies as shown in figure 1, the height of the liquid is 5 cm.

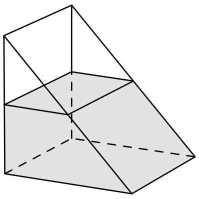


figure 1

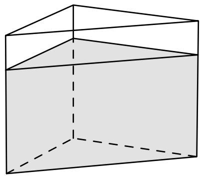


figure 2

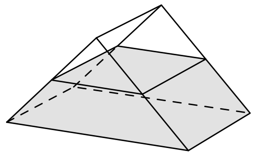


figure 3

a) What is the height of the liquid when it lies as shown in figure 2?

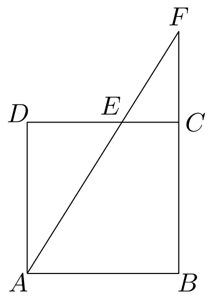
b) What is the height of the liquid when it lies as shown in figure 3?

Problem 2. A positive integer is a *palindrome* if it is written identically forwards and backwards. For example, 285582 is a palindrome. A six digit number ABCDEF, where A, B, C, D, E, F are digits, is called *cozy* if AB divides CD and CD divides EF. For example, 164896 is cozy.

Determine all cozy palindromes.

Problem 3. The square $ABCD$ has side length 1. The point E lies on the side CD. The line through A and E intersects the line through B and C at the point F.

Prove that $\dfrac{1}{|AE|^2} + \dfrac{1}{|AF|^2} = 1$.



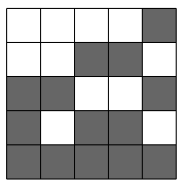
Problem 4. Georg plays the following game. He chooses two positive integers n and k. On an $n \times n$-board where all the tiles are white, Georg paints k of the tiles black. Then he counts the number of black tiles in each row, forms the square of each of these n numbers and adds up the squares. He calls the result S. In the same way he counts the number of white tiles in each row, forms the square of each of these n numbers and adds up those squares. He calls the result H. Georg would like to achieve $S - H = 49$.

Determine all possible values of n and k for which this is possible.

Example: If Georg chooses $n = 5$ and $k = 14$, he could for example paint the board as shown. Then

$$S = 1^2 + 2^2 + 3^2 + 3^2 + 5^2 = 1 + 4 + 9 + 9 + 25 = 48,$$
$$H = 4^2 + 3^2 + 2^2 + 2^2 + 0^2 = 16 + 9 + 4 + 4 + 0 = 33,$$

so in this case $S - H = 48 - 33 = 15$.



Black tiles	White tiles
1	4
2	3
3	2
3	2
5	0

Problem 5. Let $n > 2$ be an integer. The numbers $1, 2, \dots, n$ are written at the vertices of an n-gon in that order. A move consists of choosing two adjacent vertices and adding 1 to the numbers written there.

Determine all n for which it is possible to achieve that all numbers are identical after a finite number of moves.

Sponsors: Undervisningsministeriet, Novo Nordisk Fonden, Jobindex, Lundbeckfonden, Georg Mohr Fonden and Matematiklærerforeningen.