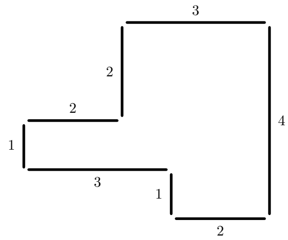
The Georg Mohr Contest 2021
Second Round

Tuesday, January 12th, 2021 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. Georg has a set of sticks. From these sticks he must create a closed figure with the property that each stick makes right angles with its neighbouring sticks. All the sticks must be used. If the sticks have the lengths 1, 1, 2, 2, 2, 3, 3 and 4, the figure might for example look like this:



(a) Prove that he can create such a figure if the sticks have the lengths 1, 1, 1, 2, 2, 3, 4 and 4.

(b) Prove that it cannot be done if the sticks have the lengths 1, 2, 2, 3, 3, 3, 4, 4 and 4.

(c) Determine whether it is doable if the sticks have the lengths 1, 2, 2, 2, 3, 3, 3, 4, 4 and 5.

Problem 2. Georg has a 4-sided die with the numbers 2, 3, 4 and 5. He rolls the die 17 times and records the result of each roll on a board, so that eventually 17 numbers are written on it. Georg notices that the average of the 17 numbers is an integer.

Is it possible that each of the numbers 2, 3, 4 and 5 appears at least three times on Georg's board?

Problem 3. Georg investigates which integers are expressible in the form

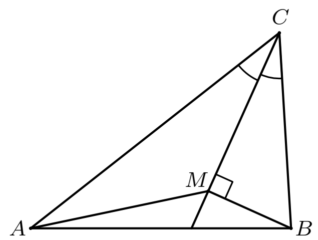
$$\pm 1^2 \pm 2^2 \pm 3^2 \pm \cdots \pm n^2.$$

For example, the number 3 can be expressed as $-1^2 + 2^2$, and the number -13 can be expressed as $+1^2 + 2^2 + 3^2 - 4^2 + 5^2 - 6^2$.

Are all integers expressible in this form?

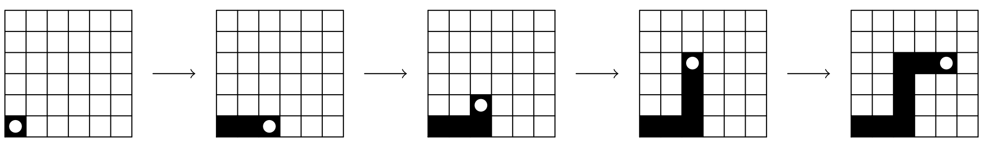
Problem 4. In triangle ABC one has $|AC| > |BC|$. The point M lies on the angular bisector of angle C, and BM is perpendicular to the angular bisector.

Prove that the area of triangle AMC is half of the area of triangle ABC.



Problem 5. A board consists of 2021×2021 squares all of which are white, except for one corner square which is black. Alma and Bertha play the following game. At the beginning, there is a piece on the black square. In each turn, the player must move the piece to a new square in the same row or column as the one in which the piece is currently. All squares that the piece moves across, including the ending square but excluding the starting square, must be white, and all squares that the piece moves across, including the ending square, become black by this move. Alma begins, and the first player unable to move loses.

Which player may prepare a strategy which secures her the victory?



The figure shows an example of possible initial moves in an equivalent game on a 6×6 board.

Sponsors: Undervisningsministeriet, Novo Nordisk Fonden, Jobindex, Lundbeckfonden, Georg Mohr Fonden and Matematiklærerforeningen.