

The Georg Mohr Contest 2017

Second Round

Tuesday, January 10th, 2017 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. A system of equations

$$\begin{aligned}x^2 ? z^2 &= -8, \\y^2 ? z^2 &= 7,\end{aligned}$$

is written on a piece of paper, but unfortunately two of the symbols are a little blurred. However, it is known that the system has at least one solution, and that each of the two question marks stands for either + or −.

What are the two symbols?

Problem 2.

Georg has a board displaying the numbers from 1 to 50.

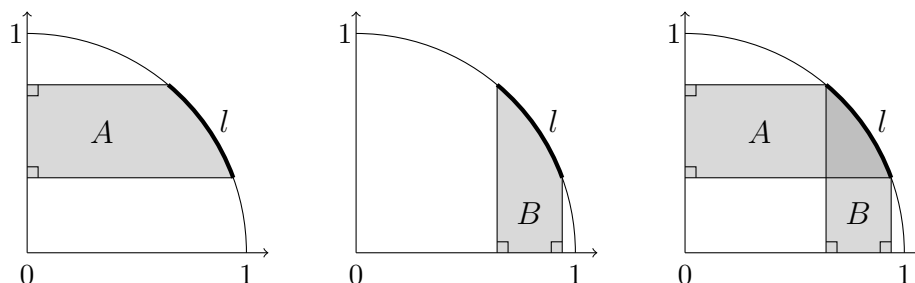
Georg may strike out a number if it can be formed by starting with the number 2 and doing one or more calculations where he either multiplies by 10 or subtracts 3.

Which of the board's numbers may Georg strike out?

| | | | | | | | | | |
|----|----|----|----|----|---------------|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |

Example: Georg may strike out 26 because it may, for example, be formed by starting with 2, multiplying by 10, subtracting 3 three times, multiplying by 10 and subtracting 3 twenty-eight times.

Problem 3. The figure shows an arc l on the unit circle and two regions A and B .



Prove that the area of A plus the area of B equals the length of l .

Problem 4. Let A, B, C og D denote the digits in a four-digit number $n = ABCD$. Determine the least n greater than 2017 satisfying that there exists an integer x such that

$$x = \sqrt{A + \sqrt{B + \sqrt{C + \sqrt{D + x}}}}$$

Problem 5. In a chess tournament, each pair of players play one game. A lost game yields 0 points, a won game yields 1 point and a tied game yields $\frac{1}{2}$ point. After the tournament, it turns out that in each group of three players, at least one got $1\frac{1}{2}$ points in the games against the two others.

What is the largest number of players that may have participated?

Sponsors: Undervisningsministeriet, Carlsbergs Mindelegat, Georg Mohr Fonden, Matematiklærerforeningen, Dansk Matematisk Forening, Gyldendal and Aarhus Universitetsforlag.