

The Georg Mohr Contest 2016

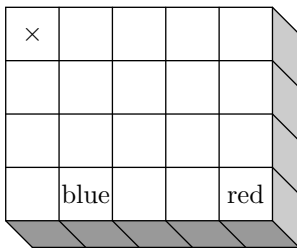
Second Round

Tuesday, January 12th, 2016 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. A class consisting of 24 students has participated in the first round of the Georg Mohr Contest, where one could obtain between 0 and 20 points. Three of the students obtained exactly the class's average. If each of the students that scored below the average had scored 4 points more, the average would have been 3 points higher.

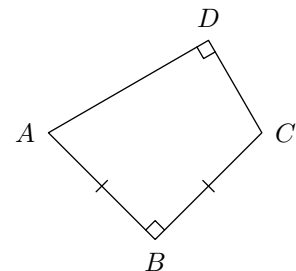
How many students scored above the class's average?



Problem 2. Twenty cubes have been coloured in the following way: There are two red faces opposite each other, two blue faces opposite each other and two green faces opposite each other. The cubes have been glued together as shown in the figure. Two faces that are glued together always have the same colour. The figure shows the colours of some of the faces.

Which colours are possible for the face marked with the symbol \times ?

Problem 3. Prove that all quadrilaterals $ABCD$ where $\angle B = \angle D = 90^\circ$, $|AB| = |BC|$ and $|AD| + |DC| = 1$, have the same area.



Problem 4. Alma and Bertha play the following game. There are 100 round, 200 triangular and 200 square pieces on a table. In each move a player must remove two pieces, but it cannot be a triangle and a square. Alma starts, and one loses if one is unable to move or if there are no pieces left when it is one's turn.

Which player has a winning strategy?

Problem 5. Find all possible values of the number

$$\frac{a+b}{c} + \frac{a+c}{b} + \frac{b+c}{a},$$

where a, b, c are positive integers, and $\frac{a+b}{c}, \frac{a+c}{b}, \frac{b+c}{a}$ are also positive integers.

Sponsors: Undervisningsministeriet, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Georg Mohr Fonden, Matematiklærerforeningen, Dansk Matematisk Forening, Gyldendal og Aarhus Universitetsforlag.