The Georg Mohr Contest 2014
Second Round

Tuesday, January 7th, 2014 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment.

Problem 1. Georg chooses three distinct digits among $1, 2, \ldots, 9$ and writes them down on three cards. When the cards are laid down next to each other, a three-digit number is formed. Georg tells his mother that the sum of the largest and the second-largest number that can be formed in this manner is 1732.

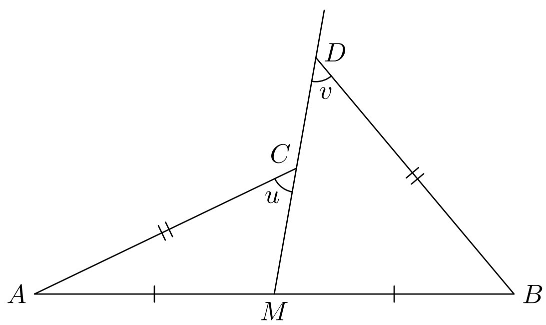
Can she figure out which three digits Georg has chosen?

Problem 2. Three gamblers play against each other for money. They each start by placing a pile of one-krone coins on the table, and from this point on the total number of coins on the table does not change. The ratio between the number of coins they start with is $6 : 5 : 4$. At the end of the game, the ratio of the number of coins they have is $7 : 6 : 5$ in some order. At the end of the game, one of the gamblers has three coins more than at the beginning.

How many coins does this gambler have at the end?

Problem 3. The points C and D lie on a halfline from the midpoint M of a segment AB, so that $|AC| = |BD|$.

Prove that the angles $u = \angle ACM$ and $v = \angle BDM$ are equal.



Problem 4. Determine all positive integers n so that both $20n$ and $5n + 275$ are perfect squares.

(A *perfect square* is a number which can be expressed as k^2, where k is an integer.)

Problem 5. Let $x_0, x_1, \ldots, x_{2014}$ be a sequence of real numbers, which for all $i < j$ satisfy $x_i + x_j \leq 2j$.

Determine the largest possible value of the sum $x_0 + x_1 + \cdots + x_{2014}$.

Sponsors: Undervisningsministeriet, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Georg Mohr Fonden, Matematiklærerforeningen, Dansk Matematisk Forening og Gyldendal.