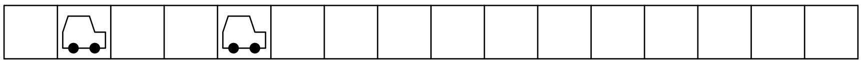
The Georg Mohr Contest 2013
Second Round

Tuesday, January 8th, 2013 at 9–13

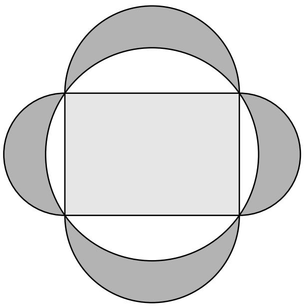
Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment.

Problem 1. The figure shows a game board with 16 squares. At the start of the game, two cars are placed in different squares. Two players A and B alternately take turns, and A starts. In each turn, the player chooses one of the cars and moves it one or more squares to the right. The left-most car may never overtake or land on the same square as the right-most car. The first player which is unable to move loses.



(a) Prove that A can win regardless of how B plays, if the two cars start as shown in the figure.

(b) Determine all starting positions in which B can win regardless of how A plays.



Problem 2. The figure shows a rectangle, its circumscribed circle and four semicircles, which have the rectangle's sides as diameters.

Prove that the combined area of the four dark gray crescent-shaped regions is equal to the area of the light gray rectangle.

Problem 3. A sequence $x_0, x_1, x_2, \ldots$ is given by $x_0 = 8$ and $x_{n+1} = \dfrac{1 + x_n}{1 - x_n}$ for $n = 0, 1, 2, \ldots$.

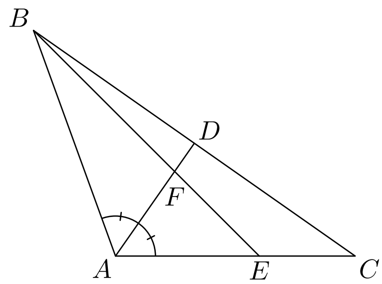
Determine the number x_{2013}.

Problem 4. The positive integer a is greater than 10, and all its digits are equal. Prove that a is not a perfect square.

(A *perfect square* is a number which can be expressed as n^2, where n is an integer.)

Problem 5. The angle bisector of A in triangle ABC intersects BC in the point D. The point E lies on the side AC, and the lines AD and BE intersect in the point F. Furthermore, $\dfrac{|AF|}{|FD|} = 3$ and $\dfrac{|BF|}{|FE|} = \dfrac{5}{3}$.

Prove that $|AB| = |AC|$.



Sponsors: Ministeriet for Børn og Undervisning, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Georg Mohr Fonden, Matematiklærerforeningen, Dansk Matematisk Forening, Gyldendal, and Texas Instruments.