

# The Georg Mohr Contest 2011

## Second Round

Thursday 20 January 9–13

Aids permitted: only writing and drawing tools.

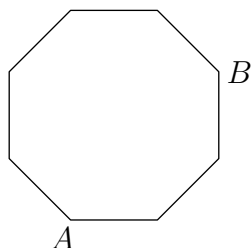
Remember that you must justify your answers.

**Problem 1.** Georg writes the numbers from 1 to 15 on different pieces of paper. He attempts to sort these pieces of paper into two stacks so that none of the stacks contains two numbers whose sum is a square number.

Prove that this is impossible.

(The square numbers are the numbers  $0 = 0^2$ ,  $1 = 1^2$ ,  $4 = 2^2$ ,  $9 = 3^2$  etc.)

**Problem 2.** In the octagon below all sides have the length 1 and all angles are equal.



Determine the distance between the corners  $A$  and  $B$ .

**Problem 3.** Determine all the ways in which the fraction  $\frac{1}{11}$  can be written as  $\frac{1}{n} + \frac{1}{m}$ , where  $n$  and  $m$  are two different positive integers.

**Problem 4.** A function  $f$  is given by

$$f(x) = x^2 - 2x.$$

Prove that there exists a number  $a$  which satisfies  $f(f(a)) = a$  without satisfying  $f(a) = a$ .

**Problem 5.** Determine all sets  $(a, b, c)$  of positive integers where one obtains  $b^2$  by removing the last digit in  $c^2$  and one obtains  $a^2$  by removing the last digit in  $b^2$ .

*Sponsors: Georg Mohr Fonden, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Dansk Matematisk Forening, Matematiklærerforeningen, Undervisningsministeriet, Gyldendal, Syddansk Universitetsforlag and Texas Instruments.*