

The Georg Mohr Contest 2011

Second Round

Thursday 20 January 9–13

Aids permitted: only writing and drawing tools.

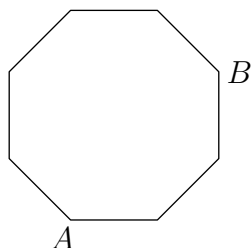
Remember that you must justify your answers.

Problem 1. Georg writes the numbers from 1 to 15 on different pieces of paper. He attempts to sort these pieces of paper into two stacks so that none of the stacks contains two numbers whose sum is a square number.

Prove that this is impossible.

(The square numbers are the numbers $0 = 0^2$, $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ etc.)

Problem 2. In the octagon below all sides have the length 1 and all angles are equal.



Determine the distance between the corners A and B .

Problem 3. Determine all the ways in which the fraction $\frac{1}{11}$ can be written as $\frac{1}{n} + \frac{1}{m}$, where n and m are two different positive integers.

Problem 4. A function f is given by

$$f(x) = x^2 - 2x.$$

Prove that there exists a number a which satisfies $f(f(a)) = a$ without satisfying $f(a) = a$.

Problem 5. Determine all sets (a, b, c) of positive integers where one obtains b^2 by removing the last digit in c^2 and one obtains a^2 by removing the last digit in b^2 .

Sponsors: Georg Mohr Fonden, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Dansk Matematisk Forening, Matematiklærerforeningen, Undervisningsministeriet, Gyldendal, Syddansk Universitetsforlag and Texas Instruments.