

The Georg Mohr Contest 2010

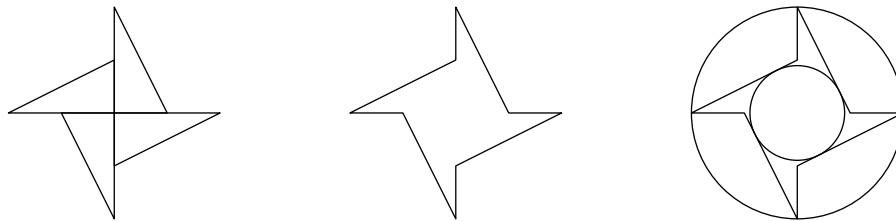
Second round

Thursday, January 7, 9:00 – 13:00

Aids permitted: Only writing and drawing tools
Remember to justify all of your answers

Problem 1. Four right triangles, each with the sides 1 and 2, are assembled to a figure as shown.

How large a fraction does the area of the small circle make up of that of the big one?

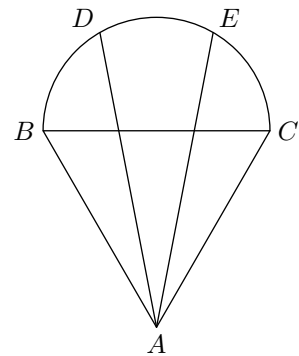


Problem 2. Prove that for every integer n there exist integers a, b, c such that $n = a^2 + b^2 - c^2$.

Problem 3. Can 29 boys and 31 girls be lined up in a row holding hands such that no one is holding hands with two girls?

Problem 4. An equilateral triangle ABC is given. With BC as diameter, a semicircle is drawn outside the triangle. On the semicircle, points D and E are chosen such that the arc lengths BD , DE and EC are equal.

Prove that the line segments AD and AE divide the side BC into three equal parts.



Problem 5. It is given that 2^{2010} is a 606-digit number beginning with 1.

How many of the numbers $1, 2, 2^2, 2^3, \dots, 2^{2009}$ begin with 4?