The Georg Mohr Contest 2009
Second round

Thursday 8 January 2009 at 9–13 hours

Tools for writing and drawing are the only ones allowed

Remember that you must give arguments for your answers

Problem 1. In the figure on the right, triangle ADE is produced from triangle ABC by a rotation by 90° about the point A. If angle D is 60° and angle E is 40° how large is then angle v?

Problem 2. Solve the system of equations

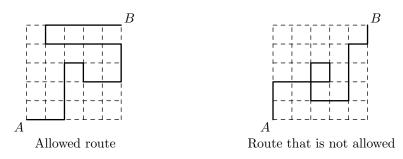$$\frac{1}{x+y} + x = 3$$
$$\frac{x}{x+y} = 2.$$

Problem 3. For a party Georg has bought a lot of chocolate creams, and when he counts how many he has he discovers that this number is a prime number. On 60 plates he places as many of the chocolates as possible divided into equal portions with one portion on each plate. He then notices that he has more than one piece left and that the number of pieces left is not a prime number.

How many pieces of chocolate has Georg left?

Problem 4. Let E be an arbitrary point different from A and B on the side AB of a square $ABCD$, and let F and G be points on the segment CE so that BF and DG are perpendicular to CE.

Prove that $DF = AG$.

Problem 5. Imagine a quadratic table which consists of $n \times n$ cells with edge length 1, where n is an arbitrary positive integer. What is the largest possible length of a route you can follow along the edges of the cells from the point A in the lower left corner to the point B in the upper right corner if you must never return to a point where you have been before? (The figure shows for $n = 5$ an example of an allowed route and an example of a route that is not allowed.)

Allowed route   Route that is not allowed