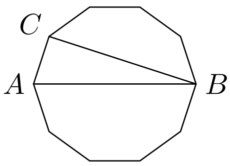
The Georg Mohr Contest 2007
Second round

Tuesday 9 January 2007 at 9–13 hours

Tools for writing and drawing are the only ones allowed

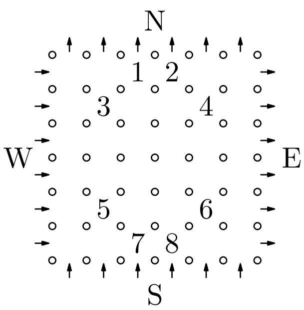


Problem 1. Triangle ABC lies in a regular decagon as shown in the figure.

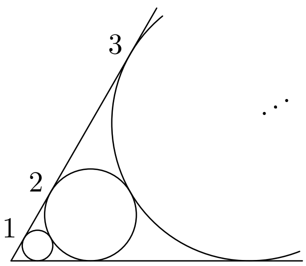
What is the ratio of the area of the triangle to the area of the entire decagon? Write the answer as a fraction of integers.

Problem 2. What is the final digit in the number 2007^{2007}?

Problem 3. A cunning dragon guards a princess. To defeat the dragon and win the princess you have to solve the following problem: The dragon has placed the numbers 1–8 in the squares between the columns in the columned hall (see figure). You are then allowed to place the numbers 9–36 in the remaining squares. The numbers 1–36 must be placed such that *any* route entering either south or west and exiting either north or east passes through at least one number from the five times table. (In the figure, north, south, east and west are denoted by N, S, E and W.)



Georg wishes to win the princess. Is it possible?



Problem 4. The figure shows a 60° angle in which are placed 2007 numbered circles (only the first three are shown in the figure). The circles are numbered according to size. The circles are tangent to the sides of the angle and to each other as shown. Circle number one has radius 1.

Determine the radius of circle number 2007.

Problem 5. The numbers $a_0, a_1, a_2, \ldots$ are determined by $a_0 = 0$ and

$$a_n = \begin{cases} 1 + a_{n-1} & \text{when } n \text{ is positive and odd,} \\ 3a_{n/2} & \text{when } n \text{ is positive and even.} \end{cases}$$

How many of these numbers are smaller than 2007?

Sponsors: Georg Mohr Fonden, Carlsbergs Mindelegat for Brygger J. C. Jacobsen, Dansk Matematisk Forening, Matematiklærerforeningen, Undervisningsministeriet, Wolfram Research, UNI-C and Gyldendal.