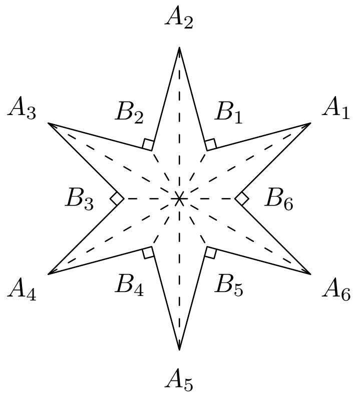
The Georg Mohr Contest 2006
Second round

Tuesday 10 January 2006 at 9-13 hours

Tools for writing and drawing are the only ones allowed

Problem 1. The star shown is symmetric with respect to each of the six diagonals shown. All segments connecting the points $A_1, A_2, \dots, A_6$ with the centre of the star have the length 1, and all the angles at $B_1, B_2, \dots, B_6$ indicated in the figure are right angles.



Calculate the area of the star.

Problem 2. Determine all triples (x, y, z) of real numbers which satisfy

$$x + y = 2 \quad \text{and} \quad xy - z^2 = 1.$$

Problem 3. A positive integer n not larger than 500 has the property that when a number m is chosen randomly from among the numbers $1, 2, 3, \dots, 499, 500$, the probability of m dividing n is $\frac{1}{100}$.

Determine the largest possible value of n.

Problem 4. From among the numbers $1, 2, 3, \dots, 2006$ are to be selected 10 different ones.

Show that one can select 10 different numbers whose sum is larger than 10039 in more ways than one can select 10 different numbers whose sum is less than 10030.

Problem 5. We consider an acute triangle ABC. The altitude from A is AD, the altitude from D in triangle ABD is DE, and the altitude from D in triangle ACD is DF.

a) Prove that the triangles ABC and AFE are similar.
b) Prove that the segment EF and the corresponding segments constructed from the vertices B og C all have the same length.

Sponsors: Georg Mohr Fonden, Carlsbergs Mindelegat for Brygger J.C.Jacobsen, Dansk Matematisk Forening, Matematiklærerforeningen, Undervisningsministeriet, Wolfram Research, UNI-C and Gyldendal.